

Tuesday, Week 2
Chapters 1.5, 2.3

Warmup

True or False: $(\forall x \in \mathbb{R})(\exists y \in \mathbb{Z})(|x - y| \leq 1/2)$

True or False: $(\forall x \in \mathbb{R})(\exists y \in \mathbb{Z})(|x - y| \leq 1/3)$

Express in quantifier notation: the set $(0, 1]$ has a maximum.

What can you say about this game of Tic-Tac-Toe if it is X 's turn to move? What if it is O 's turn to move?

O		X
X		
O	O	X

Nested Negations

Let $A = \{\text{"a"}, \text{"b"}, \text{"c"}, \dots, \text{"z"}\}$, let $D = [(x, y) \in A^2 \mid \text{"xy"} \text{ is an English word}]$.

1. Game 1: Player 1 names a letter x and Player 2 names a letter y . If xy is a word then Player 1 wins. Who has a winning strategy?
2. Game 2: Now if xy is a word then Player 2 wins. Who has a winning strategy?

Describe a counterexample:

1. All ravens are black.
2. There is a city in which all ravens are black.
3. In every country there is a city in which all ravens are black.
4. There exists a continent such that for every country in that continent there is a city in that country in which all ravens are black.
5. ...?

Uniqueness

Given: Only Superman can save us now!

1. Can Aquaman save us?
2. If Clark Kent saves us, then what does that tell us?

Prove: There is a unique real solution to $2x + 1 = 7$.

Functions

Illustrate: If $A \subset B$ then $f(A) \subset f(B)$.

Suppose $|A| = 10$, $|B| = 7$, and let $f : A \rightarrow B$ and $g : B \rightarrow A$ be functions.

1. Can f be one-to-one?
2. Can f be onto?
3. Can g be one-to-one?
4. Can g be onto?