Thursday, Week 1
Chapters 1.7-1.8

Warmup

Theorem 0.1 (Questionable Theorem) *If $x + y = 10$, then $x \neq 3$ and $y \neq 8$.*

Which of these are counterexamples?

1. When $x = 3$ and $y = 8$, $x + y = 11 \neq 10$.
2. When $x = 3$ and $y = 7$, $x + y = 10$.
3. When $x = 3$ and $y = 6$, $x + y = 9$.
4. When $x = 4$ and $y = 6$, $x + y = 10$.

Prove: Suppose n is an integer. If $3n$ is odd, then n is odd.

Proof by Cases

Define: $|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$. Prove that $|x| \geq 0$ for all x. What are the two cases?

Biconditionals and Backward Reasoning

Draw a diagram showing the relations: If it is Sunday then I am at the mall. If I am at the mall then I am shopping. If I am shopping then I have the day off work. If I have the day off work then it is Sunday.

Prove: I am at the mall if and only if I have the day off work.

It's a Trick Question

What's wrong with this proof?

Theorem 0.2 (Tricky Theorem) *If $\sqrt{2x^2-1} = x$, then $x = 1$ or $x = -1$.*

Proof:

$$\begin{aligned}
\sqrt{2x^2-1} &= x \\
2x^2 - 1 &= x^2 \\
x^2 - 1 &= 0 \\
(x-1)(x+1) &= 0 \\
x - 1 = 0 \text{ or } x + 1 &= 0 \\
x = 1 \text{ or } x &= -1
\end{aligned}$$

For which of these claims does the converse hold?

1. If $a = b$ then $a + c = b + c$.
2. If $a = b$ then $ac = bc$.
3. If $a = b$ then $a^2 = b^2$.
4. Suppose $c \neq 0$. If $a = b$ then $ac = bc$.
5. If $a^2 = 4$ then $a = 2$ or $a = -2$.
6. If $a > 0$ then $a^2 > 0$.
7. If $a > 0$ then $1/a > 0$.
8. Suppose $a, b \geq 0$. If $a > b$ then $a^2 > b^2$.