Wednesday, Week 1
Chapters 1.6-1.7

Warmup

1. Which of the following arguments are logically valid?
   (a) If we have matches, we can start a fire. We have matches. Therefore, we can start a fire.
   (b) If I did not have a liver, I would be dead. I am clearly not dead. Therefore, I have a liver.
   (c) They close the beach whenever sharks are spotted. They closed the beach. Therefore, sharks were spotted.
   (d) If I do every problem in the book I will pass the class. I will not do every problem in the book. Therefore, I will not pass the class.
   (e) If it rains then the Cubs will lose. If it does not rain then the Cubs will lose. Therefore, the Cubs will lose.
   (f) Either the Cubs or the Sox (or both) lost today. The Sox lost today. Therefore, the Cubs did not lose.
   (g) Either the Cubs or the Sox (or both) lost today. The Sox won today. Thererfore, the Cubs lost today.
2. Show that $p \Rightarrow q$ and $(\neg p) \vee q$ are equivalent.
3. Negate!
   (a) Today is either Thursday or Friday.
   (b) Albert Einstein won a Nobel Prize and two Oscars.

Counterexamples

Find counterexamples to the following claims:

1. Everybody named George is a politician.
2. If n is prime, then $2n + 1$ is prime.
3. If n is prime and odd, then either $n + 2$ or $n + 4$ is prime.
4. For any integer n, $n + 3$ is even and $n + 8$ is also even.
5. If $a + b = 0$ then $a = 0$ or $b = 0$.
6. $a \cdot b$ is zero only if $a = 0$ and $b = 0$.
7. For a number n to be divisible by 12 it is necessary for n to be divisible by 8.
8. For a number n to be divisible by 12 it is sufficient for n to be divisible by 8.

Axioms and Proofs

Which statements are "obvious" and which ones need proving?

1. If $a \cdot b = 0$ then $a = 0$ or $b = 0$.
2. For real numbers a, $a + 0 = a$.
3. For real numbers a, $a \cdot 0 = 0$.
4. If $a \leq b$ and $a \geq b$ then $a = b$.
5. If $a > 0$ and $b > 0$ then $ab > 0$.
6. The sum of two odd numbers is even.
7. $1 \neq 0$.
8. 11 is a prime number.
9. If $a > b$ then $a^2 > b^2$.
10. If $a = b$ and $b = c$ then $a = c$.

What's wrong with this proof? Find a counterexample.

Theorem 0.1 (Incorrect Theorem) *Suppose that x and y are real numbers and $x \neq 3$. If $x^2y = 9y$ then $y = 0$.*

Proof:

1. If $x^2y = 9y$, then $x^2y - 9y = 0$.
2. If $x^2y - 9y = 0$, then $(x^2 - 9)y = 0$.
3. If $(x^2 - 9)y = 0$, then $x^2 - 9 = 0$ or $y = 0$.
4. Since $x \neq 3$, $x^2 - 9 \neq 0$.
5. Since ($x^2 - 9 = 0$ or $y = 0$) but $x^2 - 9 \neq 0$, we conclude that $y = 0$.

Contraposition

State the contrapositives. Which ones sound easier to prove?

1. If n^2 is even then n is even.
2. If $a \cdot b = 0$ then $a = 0$ or $b = 0$.
3. If $a \leq b$ and $a \geq b$ then $a = b$.
4. If $ab \leq 0$ then $a \leq 0$ or $b \leq 0$.
5. If $a^2 - 2a \neq 0$ then $a \neq 2$.