14.6-7: Gradients and Critical Points

Wednesday, March 9

Gradients

The temperature at a point (x, y, z) is given by $T(x, y, z) = 200e^{-x^2-3y^2-9z^2}$ where T is in Celsius and x, y, z in meters.

1. Find the rate of change in temperature at the point $P(2, -1, 2)$ in the direction toward the point $(3, -3, 3)$.
2. In which direction does the temperature increase fastest at P?
3. Find the maximum rate of temperature increase at P.

If $L(x, y)$ is the linear approximation to a function $f(x, y)$ at a point (x_0, y_0), express L in terms of $\nabla f(x_0, y_0)$.

Also express the Chain rule in terms of the gradient.

Critical Points

Find all critical points of the following functions. Apply the Second Derivative test at those points, and use the information to sketch the graphs of the functions.

- $f(x, y) = 2x^2 - 2xy + 5y^2 - 5$

- $f(x, y) = x^3 - x - y^2$

- $f(x, y) = (x - y)(1 - xy)$