13.4,14.1: Functions of Multiple Variables

Wednesday, March 2

Partial Derivatives, Linear Approximation

Find all of the second partial derivatives of the following functions. In particular, verify that $f_{xy} = f_{yx}$.

1. $f(x, y) = \cos(xy) + xe^y$
2. $f(x, y) = \arctan(y/x)$
3. $f(x, y) = \sqrt{x^2 + y^2}$

Find a linear approximation to the function $f(x, y, z) = 3xy + xz + ye^{x^2 z}$ at $(0, 1, 0)$ and use it to approximate $f(0.1, 0.9, 0.2)$.

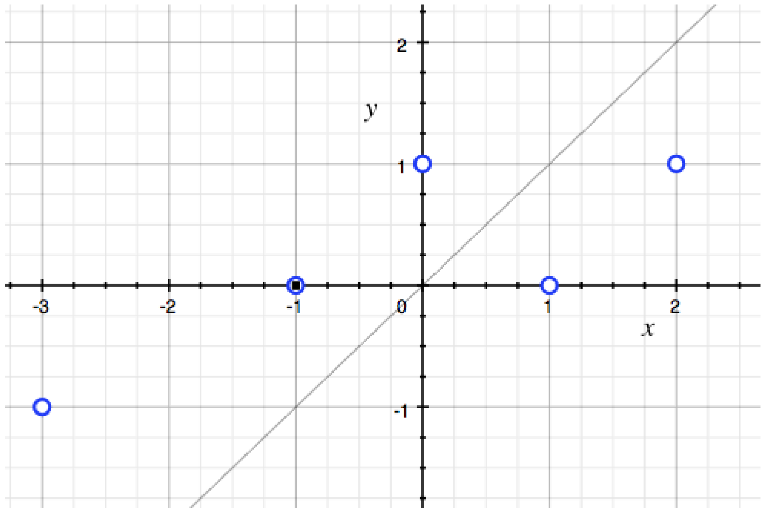
Even More Projectile Motion

A cannoneer wishes to hit a target on the ground, and to do so fires a cannonball with velocity v_0 and at angle of elevation θ. Recall that the distance the cannonball travels through the air is given by $d(v_0, \theta) = \frac{1}{g} v_0^2 \sin 2\theta$.

1. Make (rough) sketches of the graph of d for $0 \leq v \leq \sqrt{g}$ and $0 \leq \theta \leq \pi/2$.
2. Sketch a contour plot of d. If the target is a distance of 1/2 (units) from the cannoneer, sketch the curve of values (v_0, θ) that hit the target.
3. The cannoneer does not have perfect accuracy and so will have some small error in both v_0 and θ. Use a linearization of $d(v_0, \theta)$ to decide: is it a better idea (accuracywise) to fire the ball at a high angle, a low angle, or a 45-degree angle? Explain.

Best Fit Line

We would like to approximate a set of data points with a linear function. The picture below shows the five points (-3,-1), (0,1), (-1,0), (1,0), and (2,1) approximated by the function $y = x$.



1. We would like to find the values a and b that give the "best fit line" in the form $y = ax + b$. How do you think the current values $a = 1$ and $b = 0$ should be adjusted?

2. We can measure the error of the function by summing the squared distances of our predicted values from the actual y-values of the datat points: so $E(a, b) = \sum_i (ax_i + b - y_i)^2$. In our case, the error function for these five data points is $E(a, b) = 15a^2 + 5b^2 + 3 - 2ab - 2b - 10a$. Find E_a and E_b.

3. Use your previous answer: how should a and b be altered to improve the fit of the line?