13.4,14.1: Functions of Multiple Variables
Wednesday, February 24

Formulas

- $\mathbf{a} = v'\mathbf{T} + \kappa v^2 \mathbf{N}$

- $\kappa = |d\mathbf{T}|/|ds| = |\mathbf{T}'(t)|/|\mathbf{r}'(t)|$

Velocity and Acceleration

A ball attached to a stiff rod swings back and forth. The only forces acting on the ball are gravity (acting with acceleration g straight downward) and the rod itself. Suppose the ball is released from a point where the rod is parallel. Sketch the ball's position and its velocity and acceleration vectors when the rod is

1. Parallel to the ground.

2. Vertical.

3. At a 45 degree angle relative to the ground.

4. When is the ball's velocity at a maximum or minimum?

5. If T is the force exerted by the rod and m is the mass of the ball, show that $|T| \geq mg\cos\theta$ where θ is the angle between the vertical axis and the rod.

6. When is the magnitude of the force on the ball at a maximum or minimum? Can you express the magnitude of the force as a function of the ball's speed and the rod angle θ?

Contour Plots

There are three points $A(0,0)$, $B(0,1)$, $C(1,0)$ that you would like to live close to. Where should you place your house such that the sum of the *squares* of your distances from A, B, and C is minimized? If $T(x,y)$ is the sum of the three distances from your location (x,y), make a contour plot of T.

What if you just want to minimize the sum of the distances and not the sum of the squares?