13.3: Arc Length and Curvature
Wednesday, February 17

Relevant Formulas

- \( \mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} \)
- \( \mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} \)
- \( \mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t) \)
- \( \kappa = \left| \frac{d \mathbf{T}}{ds} \right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} \)

Sketches

Sketch:

1. \( x^2 - 2x + y^2 = -z^2 \)
2. \( (x + y)^2 + z^2 = 1 \)
3. The intersection of the unit sphere and the surface defined by \( (2x)^2 + (2y)^2 = 1 \)
4. \( \mathbf{r}(t) = \langle \sin t, \cos t, t \rangle \)
5. For the previous problem, pick a point and sketch \( \mathbf{T}(t), \mathbf{N}(t), \) and \( \mathbf{B}(t). \)
Arc Length and Curvature

1. Given the curve \( \mathbf{r}(t) = \langle 5 - t, 4t - 3, 3t \rangle \), find the point 4 units along the curve from the point \((4, 1, 3)\) as \(t\) increases.

2. Find formulas for the tangent and normal vectors and the curvature of the curve \( \mathbf{r}(t) = \langle t, \frac{1}{2}t^2, t^2 \rangle \).

3. Find the tangent, normal, and binormal vectors for the curve \( \mathbf{r}(t) = \langle t^2, \frac{2}{3}t^3, t \rangle \) at the point \((1, 2/3, 1)\).

Miscellany

A particle moves around the surface of a sphere. Show that its velocity vector and the vector from the center of the sphere to the particle’s position are orthogonal at all times.