13.3: Arc Length and Curvature

Wednesday, February 17

Relevant Formulas

- $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$
- $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}$
- $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$
- $\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$

Sketches

Sketch:

1. $x^2 - 2x + y^2 = -z^2$
2. $(x+y)^2 + z^2 = 1$
3. The intersection of the unit sphere and the surface defined by $(2x)^2 + (2y)^2 = 1$
4. $\mathbf{r}(t) = \langle \sin t, \cos t, t \rangle$
5. For the previous problem, pick a point and sketch $\mathbf{T}(t), \mathbf{N}(t)$, and $\mathbf{B}(t)$.

Arc Length and Curvature

1. Given the curve $\mathbf{r}(t) = \langle 5 - t, 4t - 3, 3t \rangle$, find the point 4 units along the curve from the point $(4, 1, 3)$ as t increases.

2. Find formulas for the tangent and normal vectors and the curvature of the curve $\mathbf{r}(t) = \langle t, \frac{1}{2}t^2, t^2 \rangle$.

3. Find the tangent, normal, and binormal vectors for the curve $\mathbf{r}(t) = \langle t^2, \frac{2}{3}t^3, t \rangle$ at the point $(1, 2/3, 1)$.

Miscellany

A particle moves around the surface of a sphere. Show that its velocity vector and the vector from the center of the sphere to the particle's position are orthogonal at all times.