12.5-6: Planes, Cylinders, Quadratic Surfaces

Wednesday, February 10

Warmup

Sketch the graphs of the surfaces described by the following equations:

1. $6x - 3y + 4z = 6$

2. $x^2 + y^2 = 1$ (in $\mathbb{R}^3$)

3. $x^2 + y^2 + z^2 = 1$

4. $z = x^2 + 2y^2$

5. $z = x^2 - 2y^2$

6. $z^2 = 2x^2 + y^2$

Planes

Which of the following four planes are parallel? Are any of them identical?

$$P_1 : 3x + 6y - 3z = 6, \quad P_2 : 4x - 12y + 8z = 5, \quad P_3 : 9y = 1 + 3x + 6z, \quad P_4 : z = x + 2y - 2$$

Find an equation for the plane that passes through the points $(2, 1, 2), (3, -8, 6)$, and $(-2, -3, 1)$.

Find an equation for the plane that passes through the point $(3, 1, 4)$ and contains the line of intersection of the planes $x + 2y + 3z = 1$ and $2x - y + z = -3$.

Conic sections

Consider the cone described by $z^2 = x^2 + y^2$. Sketch its intersection with the following planes. Use substitution to eliminate one of the three variables, and describe the intersection.

1. $z = 5$

2. $x = 2$

3. $z = y + 1$

4. $z = (x/2) + 1$

Find an equation for the surface consisting of all points equidistant from the points $(-1, 0, 0)$ and the plane $x = 1$. Identify the surface.