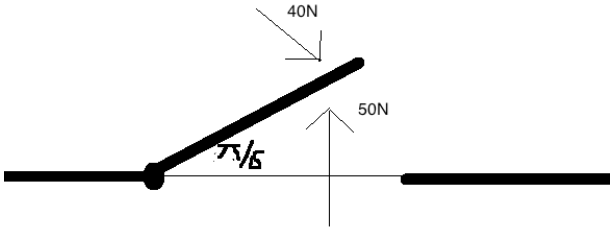
12.3: Dot Product

Wednesday, February 3

How to Open a Door

One student (pushing upward with a force of 50N) is trying to open a door and another (pushing with a force of 40N at an angle of $-\pi/4$) is trying to shut it. Which way will the door start to move?



Draw vectors a, b, c, d such that $a \cdot b > 0, a \cdot c = 0, a \cdot d < 0$.

Projections

Let $a = \langle 1, 2, 2 \rangle, b = \langle 3, -1, 0 \rangle$.

1. Find the angle between a and b.
2. Find the vector projection of b onto a.
3. Find a vector orthogonal to a.
4. Find a vector orthogonal to both a and b.
5. Are there any vectors orthogonal to a, b, and the vector from your last answer?

Work

A ball of mass m is dropped from height r. If gravity acts downward on the ball with a constant force of mg, how much work does gravity do from the time the ball is dropped until the time it hits the ground?

What if the ball is rolled down a plane with a 30° incline starting at height r?

What if the ball is at the end of a pendulum— attached to a rod of length r? How much work from the time the rod is parallel to the ground to the bottom of its swing?

Lines

Consider the line given by $ax + by = c$.

1. Show that the points on the line are the solutions to an equation of the form $u \cdot \langle x, y \rangle = k$.
2. Show that the points on the line can be expressed in the form $\{(x_0, y_0) + tv : t \in \mathbb{R}\}$ for some (x_0, y_0) and v.
3. What is the relation between u and v?