10.2/10.4: Areas, lengths, and speed

Wednesday, January 27

Warmup

1. $\sin(-3\pi/2) =$
2. $\cos(2\pi/3) =$
3. $\cos(-\theta) =$
4. $\sin(-\theta) =$
5. $\tan(\theta + \pi) =$
6. $\sin(\theta + \pi) =$
7. $u = 2x^2$; $du =$
8. $x = e^{2t}$; $dx =$
9. $y = \sqrt{x}$; $dy =$

1. Two runners start at the same spot. One runs east at 3m/s; the other runs north at 4m/s. What is the distance between the runners as a function of time?

2. A car drives with velocity $\sin(t)$ m/s for 2π seconds. How far is the car from where it started?

3. How far did the car drive?

Calculus with Parametric Curves

A cannonball is fired from ground level. As a function of the number of seconds t, its x-velocity (in m/s) is given by $dx/dt = 15$ and its y-velocity is given (approximately) by $dy/dt = 20 - 10t$.

1. What is the cannonball's speed as a function of time?
2. What is the length of the cannonball's arc through the air?
3. What is the area under the cannonball's trajectory? (You can find this with or without eliminating the parameter.)

Polar Coordinates

1. Area of a circle with radius r is:
2. Area of a sector of a circle with radius r, angle θ is:
3. Find all points of interesection of the given curves: $r = \sin\theta, r = \sin 2\theta$.
4. Find the area that lies inside both curves: $r = \sin\theta$, $r = \sin 2\theta$.
5. Which has a greater area: the four-petaled rose $r = \sin 2\theta$ or the eight-petaled rose $r = \sin 4\theta$?

Bonus

Draw pictures explaining the formulas for (1) arc lenth in parametric coordinates, (2) surface area in parametric coordinates, (3) Area enclosed by polar coordinates.