16.5-7: Surface Integrals

Wednesday, April 27

Vector Field Identities

Prove:

1. curl(curl $\mathbf{F}$) = grad(div($\mathbf{F}$)) - $\nabla^2\mathbf{F}$
2. curl($f\mathbf{F}$) = f curl($\mathbf{F}$) + $(\nabla f) \times \mathbf{F}$

Surface Area

$$A(S) = \iint_{(u,v)\in D} |\mathbf{r}_u \times \mathbf{r}_v|\, dA = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA$$

Find a parametric representation for the part of the sphere $x^2+y^2+z^2$ that lies above the cone $z = \sqrt{x^2+y^2}$.
Bonus: find its area.

Surface Integrals

$$\iint_S f(x,y,z)\,dS = \iint_D f(\mathbf{r}(u,v))|\mathbf{r}_r \times \mathbf{r}_v|\,dA$$

$$\iint_S f(x,y,z)\,dA = \iint_D f(x,y,g(x,y))\sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\,dA$$

$$\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_S \mathbf{F}\cdot\mathbf{N}\,dS = \iint_D \mathbf{F}\cdot(\mathbf{r}_u \times \mathbf{r}_v)\,dA$$

Find $\iint_S x\,dS$ where S is the triangular region with vertices $(1,0,0), (0,-2,0), (0,0,4)$.