10.1/10.5: Parametric Curves and Conic Sections

Wednesday, January 20

Welcome!

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Syllabus Recap

Homework will be assigned on Wednesdays and a quiz closely related to the homework will be given on the following Wednesday. Assignments will not be collected or graded, but solutions will normally be posted on the professor's webpage the Tuesday before the quiz and taken down about a week later.
Quizzes will normally be given on Wednesdays and will be based on the previous week's homework. Makeups are not permitted, but only your top 10 scores (out of about 12) will count toward your final grade.
For all administrative questions, consult Professor Stankova or Thomas Brown. See the course syllabus for details.

Warmup

Evaluate:

1. $(a+b)(a-b) =$
2. $\cos^2(x) + \sin^2(x) =$
3. $\tan^2(x) + 1 =$
4. $e^x \cdot e^{-x} =$
5. $\sec^2(x) - \tan^2(x) =$
6. $\dfrac{1}{3^{-2}} =$
7. $1 - \sin^2(x/2) =$
8. $(2\cos 2x)^2 + (2\sin 2x)^2 =$
9. $\sqrt{e^{4x}} =$

Make a rough sketch:

1. $(x-1)^2 + (y+2)^2 = 4$
2. $x = 3(y-2)^2 + 1$
3. $(x/3)^2 + (2(y-1))^2 = 1$

Does the function have an inverse?

1. $f(t) = 3t + 5$
2. $f(t) = t^2 - 2$
3. $f(t) = e^t$
4. $f(t) = t + 3\sin 2t$
5. $f(t) = \sqrt{t}$
6. $f(t) = t^3 - 1$

Parametric Curves

Eliminate the parameter to find a Cartesian equation of the curve. Sketch the curve and indicate with an arrow the direction in which the curve is traced.

1. $x = t^2 - 3, y = t + 2, -3 \leq t \leq 3$
2. $x = \sin(2t), y = -2\cos(2t), 0 \leq t \leq \pi$
3. $x = t^2, y = \ln t, 0 < t \leq \infty$

Determine what type of shape each equation represents, then sketch. If the shape is an ellipse or a hyperbola, express it in parametric form.

1. $x^2 + (y/3)^2 = 1$
2. $x^2/4 - y^2/9 = -1$
3. $4(x - 3)^2 = 1 - y^2$
4. $(2x + 1)^2 + y = 1$

Bonus

Prove that $\cosh^2(x) - \sinh^2(x) = 1$ for all $x \in \mathbb{R}$.

Compare the curves represented by the following parametric equations:

1. $x = t^3, y = t^2$
2. $x = t^6, y = t^4$
3. $x = e^{-3t}, y = e^{-2t}$