14.7-8: Recap

Monday, March 14

Optimization

Maximize the area of a rectangle with perimeter p: first by making a substitution to eliminate the constraint, then again by using Lagrange multipliers.

Find and classify all critical points of the function $f(x, y) = \sin x + \cos y$.

A manufacturer's production is modeled by the function $P(x, y) = 100x^{3/4}y^{1/4}$ where x is the amount of labor and y is the amount of capital. If labor costs \$200per unit, capital costs \$250 per unit, and the manager has $\$50,000$ to spend, what is the maximum attainable production level? Bonus: what is the physical interpretation of the Lagrange multiplier?

True or False?

- If $f_x(a,b)$ and $f_y(a,b)$ both exist then f is differentiable at (a,b).
- If $f(x,y) = \ln y$ then $\nabla f(x,y) = 1/y$.
- If f has a local minimum at (a,b) and f is differentiable at (a,b) then $\nabla f(a,b) = 0$.
- If $f(x,y) = \sin x + \sin y$ then $-\sqrt{2} \leq D_u f(x,y) \leq \sqrt{2}$ for all unit vectors u.
- If f is differentiable at (a,b) and $\nabla f(a,b) = 0$ then f has a local maximum or minimum at (a,b).
- If $\nabla f(a,b) = 0$, $f_{xx}(a,b) > 0$ and $f_{yy}(a,b) > 0$, then f has a local minimum at (a,b).
- If $f(x,y)$ has two local maxima then f must have a local minimum.
- If f has a single global minimum at (a,b), then the minimum of f on the unit circle occurs at the point on the circle closest to (a,b).