

14.1-2: Functions of Multiple Variables

Monday, February 29

Warmup

Find the limit, if it exists:

1. $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sqrt{x}}$

4. $\lim_{x \rightarrow 0} \frac{x}{\sin x}$

7. $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^2}$

2. $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$

5. $\lim_{x \rightarrow 0} \frac{x}{1 - \cos^2 x}$

8. $\lim_{x \rightarrow 0} \frac{\sin x - x - x^3/6}{x^5}$

3. $\lim_{x \rightarrow 0} \frac{x}{\sqrt{x+1} - 1}$

6. $\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos^2 x}$

9. $\lim_{x \rightarrow 0} \frac{x(e^x - 1)^2 \sin^3 x}{(1 - \cos^2 x)^3}$

Level Curves

Sketch the graphs of the following functions as well as their contour plots. On what domains are the functions defined?

1. $g(x, y) = \sqrt{9 - x^2 - y^2}$

3. $h(x, y) = -\ln x - \ln y$

2. $k(x, y) = \min(x, y)$

4. $f(x, y) = e^{-(x^2+y^2)/2}$

Multivariate Limits

Find the limit or show that it does not exist.

1. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{y - 2x^2}$
2. $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$
3. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 y^3}{x^6 + y^4}$
4. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + \sin^4 y}{\sin^2 x + y^4}$

True or False?

If false, give a counterexample.

1. If $f(x, y) \rightarrow L$ as $(x, y) \rightarrow (a, b)$ along every straight line through (a, b) then $\lim_{(x,y) \rightarrow (a,b)} f(x, y) = L$.
2. If f is a function then $\lim_{(x,y) \rightarrow (2,5)} f(x, y) = f(2, 5)$.
3. If $f(x, y)$ is continuous and we define $g_0(y) = f(0, y)$, then g is also continuous.
4. If $f(x, y)$ has no global maximum or minimum and $g(x) = f(0, x)$, then $g(x)$ also has no global maximum or minimum.