14.1-2: Functions of Multiple Variables

Monday, February 29

Warmup

Find the limit, if it exists:

1. $\lim_{x\to 0} \frac{e^x - 1}{\sqrt{x}}$: 1
2. $\lim_{x\to 1} \frac{x^2 - 1}{x - 1}$: 2
3. $\lim_{x\to 0} \frac{x}{\sqrt{x+1} - 1}$: 2
4. $\lim_{x\to 0} \frac{x}{\sin x}$: 1
5. $\lim_{x\to 0} \frac{x}{1 - \cos^2 x}$: 0
6. $\lim_{x\to 0} \frac{x^2}{1 - \cos^2 x}$: 2
7. $\lim_{x\to 0} \frac{\sin x - x}{x^2}$: 0
8. $\lim_{x\to 0} \frac{\sin x - x - x^3/6}{x^5}$: 1/120
9. $\lim_{x\to 0} \frac{x(e^x - 1)^2 \sin^3 x}{(1 - \cos^2 x)^3}$: 8

Level Curves

Sketch the graphs of the following functions as well as their contour plots. On what domains are the functions defined?

1. $g(x, y) = \sqrt{9 - x^2 - y^2}$: The graph is the top half of the surface of a sphere of radius 3. The domain is a circle of radius 3 and the level curves are also circles.
2. $k(x, y) = \min(x, y)$: The domain is $\mathbb{R}^2$, and the level curves make right angles at the line $y = x$.
3. $h(x, y) = -\ln x - \ln y$: The domain is $\{(x, y) : x, y > 0\}$ and the curves are hyperbolas of the form $xy = k$. As x or y approaches zero the function spikes to ∞.
4. $f(x, y) = e^{-(x^2+y^2)/2}$: A normal curve with rotational symmetry around the origin. The level curves are circles.

Multivariate Limits

Find the limit or show that it does not exist.

1. $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{y - 2x^2}$: Limit does not exist (check y and x axes)

2. $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2}$: limit does not exist (check $y = x$).

3. $\lim_{(x,y)\to(0,0)} \frac{x^3 y^3}{x^6 + y^4}$: The limit is zero.

4. $\lim_{(x,y)\to(0,0)} \frac{x^2 + \sin^4 y}{\sin^2 x + y^4}$: The limit is 1. Writing as f/g, subtract g/g and try to show that the limit of $(f - g)/g$ is zero.

True or False?

If false, give a counterexample.

1. If $f(x, y) \to L$ as $(x, y) \to (a, b)$ along every straight line through (a, b) then $\lim_{(x,y)\to(a,b)} f(x, y) = L$. False.

2. If f is a function then $\lim_{(x,y)\to(2,5) f(x,y)} = f(2, 5)$: False: this is true if f is continuous.

3. If $f(x, y)$ is continuous and we define $g_0(y) = f(0, y)$, then g is also continuous: true.

4. If $f(x, y)$ has no global maximum or minimum and $g(x) = f(0, x)$, then $g(x)$ also has no global maximum or minimum: False: try $f(x, y) = x^2 - y^2$.