

Review: True or False?

Monday, February 22

Parametric Curves, Polar Coordinates

- The curve defined by any set of parametric equations $(x, y) = (f(t), g(t))$ can also be defined by an equation of the form $y = h(x)$.
False: for example, if $x = \cos t, y = \sin t$, then the curve is a circle and so y cannot be expressed as a function of x .
- The curve defined by any equation of the form $y = h(x)$ can also be defined by a set of parametric equations $(x, y) = (f(t), g(t))$.
True: make $f(t) = t, g(t) = h(t)$.
- If $dx/dt = 0$ at some point on a curve then the tangent line at that point is horizontal.
False: First of all, if $dy/dt = 0$ as well then the particle is stopped and could have been going in any direction. Second of all, even if the particle were not stationary the tangent line would be vertical, not horizontal.
- If $dy/dt = 0$ and $dx/dt = 0$ at some point on a curve then the tangent line at that point could be horizontal, vertical, or neither.
True: think $y = x = t^3$ when $t = 0$ as an example. you have to use L'Hospital's rule to find the slope of the tangent line.
- If $f(\theta) = -f(-\theta)$ for all θ then the curve defined by $r = f(\theta)$ will have a vertical axis of symmetry.
True: informally, reason this by plotting the two points (r, θ) and $(-r, -\theta)$ and see that they are mirror images across the y -axis.

Polar Curves, Vectors

- If $x = f(t)$ and $y = g(t)$ are twice differentiable then $\frac{d^2y}{dx^2} = \frac{d^2y/dt^2}{d^2x/dt^2}$.
False: $\frac{d^2y}{dx^2} = \frac{d(dy/dt)}{dx/dt}$.
- The distance traveled by an object is equal to the integral of its velocity over time.
False: it's the integral of speed over time. The integral of velocity over time gives the displacement.
- For any vectors u and v in \mathbb{R}^n , $|u + v| \leq |u| + |v|$.
True: it's called the Triangle Inequality, as any side of a triangle is shorter than the sum of the lengths of the other two sides.
- For any $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$, $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$.
False: Say, if $\mathbf{v} = \mathbf{w}$ then the first expression is zero but the second might not be.
- For any $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$, $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{u}) = 0$.
True, since $(\mathbf{v} \times \mathbf{u})$ is perpendicular to \mathbf{u} .
- For any $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$, $\mathbf{v} \cdot (\mathbf{u} \times \mathbf{u}) = 0$.
True, since $\mathbf{v} \times \mathbf{v} = 0$

Lines and Planes

- For any line in \mathbb{R}^3 and a point not on that line, there is exactly one plane that contains both the line and the point.

True.

- For any line in \mathbb{R}^3 and a point not on that line, there is exactly one plane that is normal to the line and contains the point.

True: if the plane is described by $n \cdot x = k$, then the plane containing the point p is given by $n \cdot x = n \cdot p$

- For any two non-intersecting lines in \mathbb{R}^3 , there is exactly one plane that contains one line but not the other.

False: if the lines are parallel, there are infinitely many such planes. If the lines are skew, then no plane can contain both lines but there is only one plane that contains the first and does not *intersect* the second.

- If there is no solution t to the equation $u_0 + t\mathbf{u} = v_0 + t\mathbf{v}$ then the lines given by $\{u_0 + t\mathbf{u} : t \in \mathbb{R}\}$ and $\{v_0 + t\mathbf{v} : t \in \mathbb{R}\}$ do not intersect.

False: this just means that the two particles moving along the curves do not collide. They might hit the same spot at different times.

- The intersection of a cone and a plane is always an ellipse.

False: depending on how you slice the cone, you could get a circle, ellipse, parabola, or hyperbola.

Vector Functions

- If $|\mathbf{r}(t)| = 1$ for all t , then $|\mathbf{r}'(t)|$ is constant.

False: geometrically, the claim is that if you are moving on the surface of a sphere then your speed has to be constant. False since you could run on the surface at any speed you like.

- Different parametrizations of the same curve result in identical tangent vectors at a given point on the curve.

False: if the curve is reparametrized so that the particle moves in the opposite direction then the tangent vector will point in the opposite direction. It will, however, always be tangent to the curve at that point.