12.3-12.5: Recap

Monday, February 5

Warmup

Let $\mathbf{u} = \langle 1, 2, 3 \rangle$, $\mathbf{v} = \langle -5, 1, 1 \rangle$, $\mathbf{w} = \langle -3, 5, 7 \rangle$. Find:

1. $\mathbf{u} \cdot \mathbf{v}$
2. $\mathbf{u} \cdot \mathbf{u}$
3. $\mathbf{v} \times \mathbf{w}$
4. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$
5. What does the previous answer tell you about $\mathbf{u}, \mathbf{v}$, and $\mathbf{w}$?
6. Make a sketch of the plane $x + y + z = 1$ in the region $x, y, z \geq 0$.
7. What is the relation between the sets described by $\mathbf{u} \cdot x = 1$ and $\mathbf{u} \cdot x = 2$?

True or False

1. For any $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$, $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$.
2. For any $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$, $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$.
3. If $\mathbf{u} \cdot \mathbf{v} = 0$ then $\mathbf{u} = 0$ or $\mathbf{v} = 0$.
4. If $\mathbf{u} \times \mathbf{v} = 0$ then $\mathbf{u} = 0$ or $\mathbf{v} = 0$.
5. The intersection of two non-parallel planes is always a line.

Three Dimensions

Find a formula for the distance from a point P_0 to a line of the form $u_0 + t\mathbf{u}$. Make a picture first.

Find the set of points equidistant from two parallel lines of the form $u_0 + t\mathbf{u}$ and $u_1 + t\mathbf{u}$. Make a sketch first and guess what the answer should be before doing any computations.

Bonus: What if the lines intersect? What if they are skew lines?

Given two intersecting planes described by the equations $\mathbf{u} \cdot x = k_1$ and $\mathbf{v} \cdot x = k_2$, find a way to describe the intersection.