Let $u = (1, 1), v = (-3/2, 2)$. Find and plot:

1. $u + v$: $(-1, 2, 3)$
2. $2u - v$: $(7/2, 0)$
3. $u/|u|$: $(\sqrt{2}/2, \sqrt{2}/2)$
4. $\frac{1}{3}u + \frac{2}{3}v = (-2, 3, 5/3)$. Also $2/3$ of the way along the line segment from $u$ to $v$.
5. $-v/|v|$: $(3/5, -4/5)$
6. A unit vector perpendicular to $v$: $\pm (3/5, 4/5)$

Write at least 3 tips for plotting points in polar coordinates. Use your tips to plot the curve $r = \sin \theta + \cos^2 \theta$.

- Find $dr/d\theta$. When $dr/d\theta = 0$, the curve’s distance from the origin is at a local maximum (or minimum). You can also find the intervals on which $r$ is increasing or decreasing.
- Find any angles $\theta$ where $r = 0$. If your function is differentiable, the lines at these angles will lie tangent to the curve at the origin.
- Maybe plot some simple points, like $\theta = 0$ or $\theta = \pi$. 

\[ r = \sin \theta + \cos^2 \theta \]
Set up the integral that would give you the length of this curve for \(0 \leq \theta \leq 2\pi\). Draw a picture to help you remember the arc length formula for polar coordinates. The picture is a triangle after a small bit of progress \(\Delta \theta\). One side (the tangential axis) has progressed \(r\Delta \theta\) and the other (the radial axis) has progressed \(\Delta r\). The total length \(\Delta s\) is the hypotenuse, so

\[
s = \int ds = \int_0^{2\pi} \sqrt{(r\Delta \theta)^2 + \Delta r^2} d\theta.
\]

Let \(u = (1, 1), v = (-3, 1), w = (-1, 3)\). Find numbers \(\alpha, \beta\) such that \(w = \alpha u + \beta v\) and plot your result. The solution is \(2u + v = w\).

A 300lb football player running east tackles a 200lb football player running south. If the second player was running twice as fast as the first player and they fall in the same direction post-tackle, what vector describes that direction? (Physics fact: the total momentum of the players, equal to mass times velocity, is conserved.) Represent the momentum vector for player 1 as \(\langle 300, 0 \rangle\) and the momentum vector for player 2 as \(2 \cdot \langle 0, -200 \rangle\), so the combined momentum is \(\langle 300, -400 \rangle = 100\langle 3, -4 \rangle\).

There are two objects: one of mass \(M\) at location \(A\) and one of mass \(m\) at location \(B\). Where is the center of mass of the system? (Imagine the center of mass as the fulcrum of a scale balancing the two objects.) The center of mass is at \(\frac{AM + Bm}{M + m}\), which is on the line segment between \(A\) and \(B\) since \(M/(M + m) + m/(M + m) = 1\).

True or False?

1. The polar curves \(r = 1 - \sin 2\theta, r = \sin 2\theta - 1\) have the same graph.
   True.

2. If \(x = f(t)\) and \(y = g(t)\) are twice differentiable, then \(\frac{d^2 y}{dx^2} = \frac{d^2 y/dt^2}{d^2 x/dt^2}\).
   False.

3. The distance traveled by an object is equal to the integral of its velocity over time.
   False...it’s the integral of speed over time.
4. For any vectors $u$ and $v$ in $\mathbb{R}^n$, $u + v = v + u$. True.

5. For any vectors $u$ and $v$ in $\mathbb{R}^n$, $|u + v| = |u| + |v|$. False, unless the vectors are pointing in the same direction.

6. The set of points $\{x, y, z | x^2 + y^2 = 1\}$ is a circle. False: it’s an infinite cylinder.