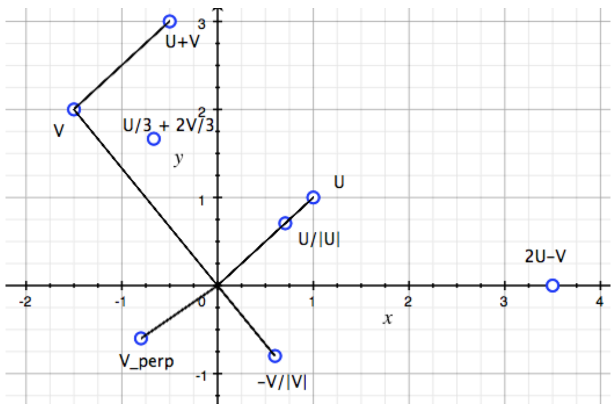
10.2, 12.2: Recap

Monday, February 1

♣

Let $u = \langle 1, 1\rangle, v = \langle -3/2, 2\rangle$. Find and plot:

1. $u + v$: $\langle -1, 2, 3\rangle$
2. $2u - v$: $\langle 7/2, 0\rangle$
3. $u/|u|$: $\langle \sqrt{2}/2, \sqrt{2}/2\rangle$
4. $\frac{1}{3}u + \frac{2}{3}v = \langle -2, 3, 5/3\rangle$. Also 2/3 of the way along the line segment from u to v.
5. $-v/|v|$: $\langle 3/5, -4/5\rangle$
6. A unit vector perpendicular to v: $\pm\langle 3/5, 4/5\rangle$



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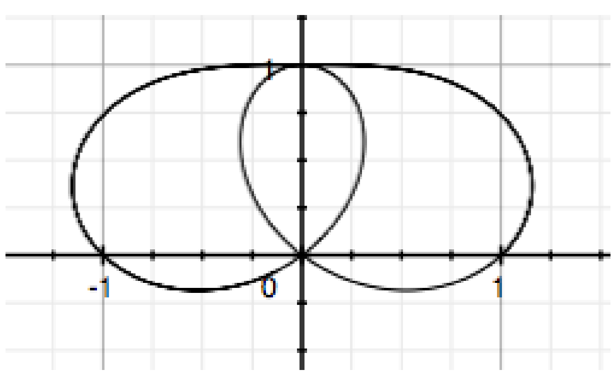
Write at least 3 tips for plotting points in polar coordinates. Use your tips to plot the curve $r = \sin\theta + \cos^2\theta$.

- Find $dr/d\theta$. When $dr/d\theta = 0$, the curve's distance from the origin is at a local maximum (or minimum). You can also find the intervals on which r is increasing or decreasing.
- Find any angles θ where $r = 0$. If your function is differentiable, the lines at these angles will lie tangent to the curve at the origin.
- Maybe plot some simple points, like $\theta = 0$ or $\theta = \pi$.



Set up the integral that would give you the length of this curve for $0 \leq \theta \leq 2\pi$. Draw a picture to help you remember the arc length formula for polar coordinates.
The picture is a triangle after a small bit of progress $\Delta\theta$. One side (the tangential axis) has progessed $r\Delta\theta$ and the other (the radial axis) has progressed Δr. The total length Δs is the hypotenuse, so

$$s = \int ds = \int \sqrt{(rd\theta)^2 + dr^2} = \int_{\theta=0}^{2\pi} \sqrt{r^2 + (dr/d\theta)^2} d\theta.$$

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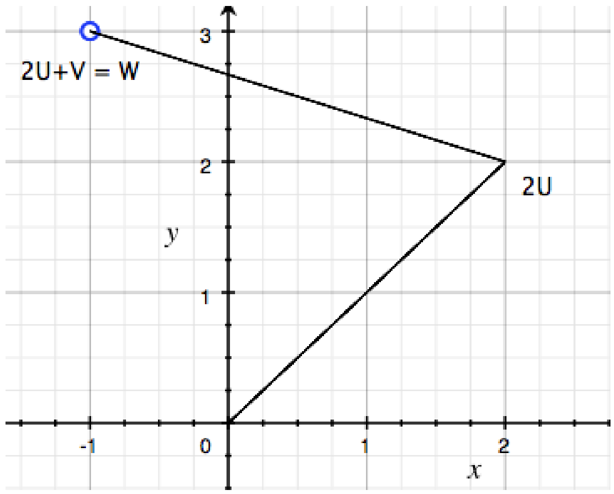
Let $u = \langle 1, 1\rangle, v = \langle -3, 1\rangle, w = \langle -1, 3\rangle$. Find numbers α, β such that $w = \alpha u + \beta v$ and plot your result.
The solution is $2u + v = w$.



A 300lb football player running east tackles a 200lb football player running south. If the second player was running twice as fast as the first player and they fall in the same direction post-tackle, what vector describes that direction? (Physics fact: the total momentum of the players, equal to mass times velocity, is conserved.)
Represent the momentum vector for player 1 as $\langle 300, 0\rangle$ and the momentum vector for player 2 as $2\cdot\langle 0, -200\rangle$, so the combined momentum is $\langle 300, -400\rangle = 100\langle 3, -4\rangle$.

There are two objects: one of mass M at location A and one of mass m at location B. Where is the center of mass of the system? (Imagine the center of mass as the fulcrum of a scale balancing the two objects.)
The center of mass is at $\dfrac{AM + Bm}{M + m}$, which is on the line segment between A and B since $M/(M + m) + m/(M + m) = 1$.

♠ True or False?

1. The polar curves $r = 1 - \sin 2\theta$, $r = \sin 2\theta - 1$ have the same graph.

   True.

2. If $x = f(t)$ and $y = g(t)$ are twice differentiable, then $\frac{d^2y}{dx^2} = \frac{d^2y/dt^2}{d^2x/dx^2}$.

   False.

3. The distance traveled by an object is equal to the integral of its velocity over time.

   False. . . it's the integral of speed over time.

4. For any vectors u and v in $\mathbb{R}^n$, $u + v = v + u$. True.
5. For any vectors u and v in $\mathbb{R}^n$, $|u + v| = |u| + |v|$. False, unless the vectors are pointing in the same direction.
6. The set of points $\{x, y, z | x^2 + y^2 = 1\}$ is a circle. False: it's an infinite cylinder.