15.8-15.9: Triple Integrals, Change of Variables

Monday, April 11

Spherical Coordinates

Find the volume of the solid E that lies above the cone $z = \sqrt{x^2 + y^2}$ and inside the sphere with boundary $x^2 + y^2 + z^2 = 1$. What is the ratio of this volume to the volume of the sphere? Make an estimate before finding the answer.

Find the volume of the smaller wedge cut from a sphere of radius a by two planes that intersect along a diameter at an angle of $\pi/6$.

True or False?

1. If a particle moves around on the surface of a sphere with $d\phi/dt$ and $d\theta/dt$ constant, then the speed of the particle is constant.

2. If a particle has fixed coordinates ϕ and θ but moves with $d\rho/dt$ constant, then the speed of the particle is constant.

3. $\displaystyle\int_{y=1}^{4}\int_{x=0}^{1}(x^2+\sqrt{y})\sin(x^2y^2)\,dx\,dy \le 9.$

4. Every point in $\mathbb{R}^3$ is *uniquely* represented by a set of spherical coordinates (ρ,θ,ϕ).

5. $\displaystyle\int_0^1\int_0^x\sqrt{x+y^2}\,dy\,dx = \int_0^x\int_0^1\sqrt{x+y^2}\,dx\,dy$

6. When $f(x,y,z)=1$, the integral $\iiint_V f(x,y,z)\,dx\,dy\,dz$ gives the volume of the region V.

Change of Variables

Evaluate the following integral by making the change of coordinates $u=3x, v=2y$:

$$\iint_R \sin(9x^2+4y^2)\,dA$$

where R is the region in the first quadrant bounded by the ellipse $9x^2+4y^2=1$.