Midterm 2: Practice Test

Monday, April 4

Problem 1

Determine, with proof, whether each of the following functions is continuous at the origin:

1. $f(x, y) = \begin{cases} \frac{2xy}{x^2+y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}.$

2. $g(x, y) = \begin{cases} \frac{2xy^2}{x^2+y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}.$

Problem 2

A particle is at position $\langle 3.02, 1.97, 5.99 \rangle$. Use a linear approximation to estimate its distance from the origin.

Problem 3

Find the maximum and minimum values of the function $f(x, y) = x^2 + 2xy - 2x - 2y + y^2$ given the constraints $x^2 + y^2 = 1$.

Problem 4

Find the volume of the solid above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

Problem 5

Find and classify all critical points of the function $f(x, y) = x^3 + xy + y^2$.

True or False?

1. If a function f has a single global maximum at (a, b) then $\nabla f(x, y)$ points along the line segment from (x, y) to (a, b).
2. For any unit vector $\mathbf{u}$ and any point $\mathbf{a}$, $Df_{-\mathbf{u}}(\mathbf{a}) = -Df_{\mathbf{u}}(\mathbf{a})$.
3. If f_x and f_y exist and are continuous in a neighborhood around (a, b) then f is differentiable at (a, b).
4. If f has a unique global maximum at a point $\mathbf{a}$ then the maximum value of f on a domain D occurs at the point in D closest to $\mathbf{a}$.
5. There exists a function f with continuous second-order partial derivatives such that $f_x(x, y) = x + y^2$ and $f_y(x, y) = x - y^2$.
6. $f_y(a, b) = \lim_{y \to b} \dfrac{f(a, y) - f(a, b)}{y - b}$.
7. If f and g are both differentiable, then $\nabla(fg) = f\nabla g + g\nabla f$.
8. If $\nabla f(x, y) = \lambda \nabla g(x, y)$ for some λ then x is an extreme value of f on the set $\{(a, b) : g(a, b) = g(x, y)\}$.
9. If $f(x, y) = f(y, x)$ for all $x, y \in \mathbb{R}$ then $\displaystyle\int_{x=0}^{a} \int_{y=0}^{b} f(x, y)\, dy\, dx = \int_{x=0}^{b} \int_{y=0}^{a} f(x, y)\, dy\, dx$.
10. For any integrable function f, $\displaystyle\int_{x=0}^{a} \int_{y=x}^{a} f(x, y)\, dx\, dy = \int_{y=0}^{a} \int_{x=y}^{a} f(x, y)\, dx\, dy$.