Review 2: Many True/False

Wednesday, May 4

1. If $\mathbf{F}$ is a vector field then $\nabla \cdot \mathbf{F}$ is a vector field.
2. If $\mathbf{F}$ is a vector field then $\nabla \times \mathbf{F}$ is a vector field.
3. I f has continuous partial derivatives on $\mathbb{R}^3$ then $\nabla \cdot (\nabla \times f) = 0$.
4. If f has continuous partial derivatives on $\mathbb{R}^3$ and C is any circle then $\int_C \nabla f \cdot d\mathbf{r} = 0$.
5. If $\mathbf{F} = \langle P, Q \rangle$ and $P_y = Q_x$ in an open region D then $\mathbf{F}$ is conservative.
6. If $\mathbf{F}$ and $\mathbf{G}$ are vector fields and $\nabla \times \mathbf{F} = \nabla \times \mathbf{G}$ then $\mathbf{F} = \mathbf{G}$.
7. The work done by a conservative force field in moving a particle around a closed path is zero.
8. There is a vector field $\mathbf{F}$ such that $\nabla \times \mathbf{F}\langle x, y, z \rangle$.
9. If $\mathbf{F}$ is conservative then $\nabla \times \mathbf{F} = 0$.
10. If $\mathbf{F}$ is conservative then $\nabla \cdot \mathbf{F} = 0$.
11. If $\nabla \times \mathbf{F} = 0$ then $\mathbf{F}$ is conservative.
12. Green's Theorem is just the Divergence Theorem in two dimensions.
13. curl(div($\mathbf{F}$)) is not a meaningful expression.
14. The integral $\displaystyle\int_{\phi=0}^{\pi/2} \int_{\theta=0}^{\pi/2} \int_{\rho=0}^{1} \rho^2 \sin\theta \, d\rho \, d\theta \, d\phi$ gives the volume of 1/4 of a sphere.
15. $\displaystyle\int_{r=-1}^{1} \int_{\theta=0}^{1} e^{r^2+\theta^2} \, d\theta \, dr = \left[\int_{r=-1}^{1} e^{r^2} \, dr\right] \left[\int_{\theta=0}^{1} e^{\theta^2} \, d\theta\right]$
16. If C is a closed curve then $\int_C f \, ds = 0$ for any function f.
17. If $\int_C f \, ds = 0$ then C is a closed curve.
18. If the work done by a force $\mathbf{F}$ on an object moving along a curve is W, then if the object moves along the curve in the opposite direction the work done by $\mathbf{F}$ will be $-W$.
19. If a particle moves along a curve C, the total work done by a force $\mathbf{F}$ on the object is independent of how quickly the particle moves.
20. If a force points only in the x direction then the work done by the force on a particle depends only on the particle's starting and ending x-positions.
21. If ∇f exists everywhere then f is continuous everywhere.
22. If $f_x = f_y = 0$ at a point (x, y) then f is differentiable at (x, y).
23. If f is differentiable along every straight line going through a point (x, y) then f is differentiable at (x, y).
24. For any x, $f(x - \nabla f(x)) \leq f(x)$.
25. If $f(x, y) = 1$ then $\iint_D f(x, y) \, dA$ is equal to the area of the domain D.
26. For any $a, b \in \mathbb{R}$ and continuous function f, $\int_{x=0}^{a} \int_{y=0}^{b} f(x, y) \, dy \, dx = \int_{y=0}^{b} \int_{x=0}^{a} f(x, y) \, dx \, dy$.

27. If $f(x,y) = g(x)h(y)$, then $\iint_D f(x,y)\,dA = \left(\iint_D g(x)\,dA\right)\left(\iint_D h(y)\,dA\right)$.

28. If $f_{xx} > 0$ and $f_{yy} > 0$ at a point (x,y) then the point (x,y) is a local minimum of the function f.

29. If (x,y) is a local minimum of a function f then f is differentiable at (x,y) and $\nabla f(x,y) = 0$.

30. If $\nabla f(x,y) = 0$ then (x,y) is a local minimum or maximum of f.

31. If $f_{xx} > 0$ and $f_{yy} < 0$ at a point (x,y) then (x,y) is a saddle point of f.

32. If ∇f is never zero then the minimum and maximum of f on a closed and bounded domain D must occur on the boundary.

33. If f has a critical point in the interior of a closed and bounded domain D then the minimum and maximum of f on D occur in the interior of D.

34. If x is a minimum of f given the constraints $g(x) = h(x) = 0$ then $\nabla f(x) = \lambda \nabla g(x)$ and $\nabla f(x) = \mu \nabla h(x)$ for some scalars λ and μ.

35. A region D simply connected if any two points in D can be joined by a curve that stays inside D.

36. If $\mathbf{F}$ is conservative on D and $\mathbf{F} = P\mathbf{i} + Q\mathbf{j}$ (where P and Q are continuously differentiable) then $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ throughout D.

37. If P and Q are continuously differentiable and $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ throughout D then $\mathbf{F}$ is conservative on D.

38. If there exists a closed curve C in D such that $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ then $\mathbf{F}$ is conservative on D.

39. If $\mathbf{F}$ is conservative on a region D then there is some function f on D such that $\nabla f = \mathbf{F}$.

40. If two triangles share an edge then the work a force field does on a particle traveling along the two triangles one at a time is the same as if the particle traveled along the quadrilateral boundary of the union of the two triangles.

41. The kinetic energy of an object plus its potential energy due to gravity is always constant.

42. $\int_{-C} f\,ds = -\int_C f\,ds$

43. If a particle travels in a closed loop then the total work done on the particle over the loop is zero.

44. If we have a region S in (u,v)-space and form a region R by the transformation $x = 2u + v, y = u - 2v$, then the area of R is $1/5$ the area of S.

45. If we instead apply the transformation $x = 2u + v, y = 4u + 2v$ then the region of R is zero.

46. If a particle is moving in a constant force field then the work done on the particle is proportional to the distance the object travels.

47. If a particle is moving in a constant force field then the work done on the particle is proportional to the particle's distance from its starting position.

48. If a particle moves around on the surface of a sphere with $d\phi/dt$ and $d\theta/dt$ constant, then the speed of the particle is constant.

49. If a particle has fixed coordinates ϕ and θ but moves with $d\rho/dt$ constant, then the speed of the particle is constant.

50. $\displaystyle\int_{y=1}^{4} \int_{x=0}^{1} (x^2 + \sqrt{y}) \sin(x^2 y^2)\,dx\,dy \leq 9.$

51. Every point in $\mathbb{R}^3$ is *uniquely* represented by a set of spherical coordinates (ρ, θ, ϕ).

52. $\displaystyle\int_0^1 \int_0^x \sqrt{x+y^2}\, dy\, dx = \int_0^x \int_0^1 \sqrt{x+y^2}\, dx\, dy$

53. When $f(x,y,z) = 1$, the integral $\iiint_V f(x,y,z)\, dx\, dy\, dz$ gives the volume of the region V.

54. If a function f has a single global maximum at (a,b) then $\nabla f(x,y)$ points along the line segment from (x,y) to (a,b).

55. For any unit vector $\mathbf{u}$ and any point $\mathbf{a}$, $Df_{-\mathbf{u}}(\mathbf{a}) = -Df_{\mathbf{u}}(\mathbf{a})$.

56. If f_x and f_y exist and are continuous in a neighborhood around (a,b) then f is differentiable at (a,b).

57. If f has a unique global maximum at a point $\mathbf{a}$ then the maximum value of f on a domain D occurs at the point in D closest to $\mathbf{a}$.

58. There exists a function f with continuous second-order partial derivatives such that $f_x(x,y) = x + y^2$ and $f_y(x,y) = x - y^2$.

59. $\displaystyle f_y(a,b) = \lim_{y \to b} \frac{f(a,y) - f(a,b)}{y-b}$.

60. If f and g are both differentiable, then $\nabla(fg) = f\nabla g + g\nabla f$.

61. If $\nabla f(x,y) = \lambda \nabla g(x,y)$ for some λ then x is an extreme value of f on the set $\{(a,b) : g(a,b) = g(x,y)\}$.

62. If $f(x,y) = f(y,x)$ for all $x, y \in \mathbb{R}$ then $\displaystyle\int_{x=0}^a \int_{y=0}^b f(x,y)\, dy\, dx = \int_{x=0}^b \int_{y=0}^a f(x,y)\, dy\, dx$.

63. For any integrable function f, $\displaystyle\int_{x=0}^a \int_{y=x}^a f(x,y)\, dx\, dy = \int_{y=0}^a \int_{x=y}^a f(x,y)\, dx\, dy$.

64. If $f_x(a,b)$ and $f_y(a,b)$ both exist then f is differentiable at (a,b).

65. If $f(x,y) = \ln y$ then $\nabla f(x,y) = 1/y$.

66. If f has a local minimum at (a,b) and f is differentiable at (a,b) then $\nabla f(a,b) = 0$.

67. If $f(x,y) = \sin x + \sin y$ then $-\sqrt{2} \le D_u f(x,y) \le \sqrt{2}$ for all unit vectors u.

68. If f is differentiable at (a,b) and $\nabla f(a,b) = 0$ then f has a local maximum or minimum at (a,b).

69. If $\nabla f(a,b) = 0$, $f_{xx}(a,b) > 0$ and $f_{yy}(a,b) > 0$, then f has a local minimum at (a,b).

70. If $f(x,y)$ has two local maxima then f must have a local minimum.

71. If f has a single global minimum at (a,b), then the minimum of f on the unit circle occurs at the point on the circle closest to (a,b).

72. If $f(x,y) \to L$ as $(x,y) \to (a,b)$ along every straight line through (a,b) then $\lim_{(x,y)\to(a,b)} f(x,y) = L$.

73. If f is a function then $\lim_{(x,y)\to(2,5) f(x,y)} = f(2,5)$.

74. If $f(x,y)$ is continuous and we define $g_0(y) = f(0,y)$, then g is also continuous.

75. If $f(x,y)$ has no global maximum or minimum and $g(x) = f(0,x)$, then $g(x)$ also has no global maximum or minimum.

76. For any $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$, $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$.

77. For any $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$, $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$.

78. If $\mathbf{u} \cdot \mathbf{v} = 0$ then $\mathbf{u} = 0$ or $\mathbf{v} = 0$.

79. If $\mathbf{u} \times \mathbf{v} = 0$ then $\mathbf{u} = 0$ or $\mathbf{v} = 0$.

80. The intersection of two non-parallel planes is always a line.

81. The polar curves $r = 1 - \sin 2\theta$, $r = \sin 2\theta - 1$ have the same graph.

82. If $x = f(t)$ and $y = g(t)$ are twice differentiable, then $\frac{d^2y}{dx^2} = \frac{d^2y/dt^2}{d^2x/dx^2}$.

83. The distance traveled by an object is equal to the integral of its velocity over time.

84. For any vectors u and v in $\mathbb{R}^n$, $u + v = v + u$.

85. For any vectors u and v in $\mathbb{R}^n$, $|u + v| = |u| + |v|$.

86. The set of points $\{x, y, z | x^2 + y^2 = 1\}$ is a circle.

87. The curve defined by any set of parametric equations $(x, y) = (f(t), g(t))$ can also be defined by an equation of the form $y = h(x)$.

88. The curve defined by any equation of the form $y = h(x)$ can also be defined by a set of parametric equations $(x, y) = (f(t), g(t))$.

89. If $dy/dt = 0$ at some point on a curve then the tangent line at that point is horizontal.

90. If a circle is parametrized as $(x, y) = (\cos t, \sin t)$, then for any t the angle between $(x(t), y(t))$ and the positive x-axis will be equal to t.

91. If $f(\theta) = f(-\theta)$ for all θ, then the curve defined by $r = f(\theta)$ will have a vertical axis of symmetry.

92. If $f(\theta) = f(\theta + \pi)$ for all θ, then the curve defined by $r = f(\theta)$ will be unchanged when it is rotated by 180 degrees about the origin.

93. If the force on a particle is always perpendicular to the particle's velocity then the particle will never change speed.

94. If the force on a particle is always parallel to the particle's velocity then the particle will never change direction.