14.5-6: Chain Rule, Partial Derivatives

Friday, March 4

Recap

Consider the function $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$.

Find f_x and f_y at the point $(0, 0)$ [hint: what are the values of f along the x and y axes?] and use them to approximate $f(0.1, -0.1)$. How good of an approximation is it to $f(0.1, -0.1)$?

Do the same with the function $g(x, y) = x^2 + y^2$. Why is the quality of your two approximations so different?

Chain Rule

Define $f(x, y) = ye^x$. If $y = t^2$ and $x = \sqrt{t}$, find $\frac{\partial f}{\partial t}$ when $t = 1$.

If $g(x, y, z) = xyz$ and $x = t, y = t, z = t^2$, find $\frac{\partial g}{\partial t}$ at $t = 2$. What does this have to do with the power rule? What about the product rule?

Implicit Differentiation

Find $\partial z/\partial x$ and $\partial z/\partial y$ for an arbitrary point on the surface $x^2 + 2y^2 + 3z^2 = 1$.

Do the same for the surface defined by the equation $e^z = xyz$.

Directional Derivatives

Say we want to minimize the function $f(x, y) = x^2 + 2y^2 + xy + 7x$ and we are currently sitting at the point $(0, 0)$. Using the limit definition of a directional derivative, find the derivative of f in the direction $\mathbf{v} = \langle -\sqrt{2}/2, \sqrt{2}/2 \rangle$.

Will the function decrease faster if we head in the direction $\langle -1, 1 \rangle$ or $\langle -3/5, 4/5 \rangle$?