Contour Plots

Sketch contour plots of the following functions. Locate local maxima and minima, or determine that there are none. Also sketch the graphs of the functions.

1. \( f(x, y) = (x - 3)^2 + 2(y + 1)^2 \)
2. \( f(x, y) = x^2 - y^2 \)
3. \( f(x) = \langle 2, 1 \rangle \cdot x \)
4. \( f(x, y) = y/\sqrt{x^2 + y^2} \)

The Unit Ball

Consider the three functions \( f(x, y) = \max(|x|, |y|) \), \( g(x, y) = \sqrt{x^2 + y^2} \), and \( h(x, y) = |x| + |y| \). Sketch one or two level sets of the the three functions on the same plot. Come up with a conjecture about the relation between \( f \), \( g \), and \( h \).
Limits and Continuity

Find the limit or show that it does not exist:

1. \( \lim_{(x,y) \to (1,2)} (5x^3 - x^2y^2) \)
2. \( \lim_{(x,y) \to (0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2} \)
3. \( \lim_{(x,y) \to (0,0)} \frac{y^2 \sin^2 x}{x^4 + y^4} \)
4. \( \lim_{(x,y) \to (0,0)} \frac{xy}{\sqrt{x^2 + y^2}} \)

Determine the set of points on which the function is continuous:

1. \( G(x, y) = \ln(x^2 + y^2 - 4) \)
2. \( f(x, y, z) = \arcsin(x^2 + y^2 + z^2) \)
3. \( f(x, y) = \begin{cases} \frac{x^3y^3}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 1 & (x, y) = (0, 0) \end{cases} \)