

## 14.1-2: Functions of Multiple Variables

Friday, February 26

### Contour Plots

Sketch contour plots of the following functions. Locate local maxima and minima, or determine that there are none. Also sketch the graphs of the functions.

1.  $f(x, y) = (x - 3)^2 + 2(y + 1)^2$

2.  $f(x, y) = x^2 - y^2$

3.  $f(\mathbf{x}) = \langle \mathbf{2}, \mathbf{1} \rangle \cdot \mathbf{x}$

4.  $f(x, y) = y/\sqrt{x^2 + y^2}$

### The Unit Ball

Consider the three functions  $f(x, y) = \max(|x|, |y|)$ ,  $g(x, y) = \sqrt{x^2 + y^2}$ , and  $h(x, y) = |x| + |y|$ . Sketch one or two level sets of the the three functions on the same plot. Come up with a conjecture about the relation between  $f$ ,  $g$ , and  $h$ .

## Limits and Continuity

Find the limit or show that it does not exist:

1.  $\lim_{(x,y) \rightarrow (1,2)} (5x^3 - x^2y^2)$

2.  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2}$

3.  $\lim_{(x,y) \rightarrow (0,0)} \frac{y^2 \sin^2 x}{x^4 + y^4}$

4.  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{\sqrt{x^2 + y^2}}$

Determine the set of points on which the function is continuous:

1.  $G(x, y) = \ln(x^2 + y^2 - 4)$

2.  $f(x, y, z) = \arcsin(x^2 + y^2 + z^2)$

3.  $f(x, y) = \begin{cases} \frac{x^2 y^3}{2x^2 + y^2} & (x, y) \neq (0, 0) \\ 1 & (x, y) = (0, 0) \end{cases}$