13.4: Velocity and Acceleration

Friday, February 19

Some Formulas

- $v = \mathbf{v}$
- $\mathbf{a} = v'\mathbf{T} + \kappa v^2 \mathbf{N}$
- $\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|}$

More Quadratic Surfaces

Classify and sketch the surface:

1. $x^2 + z^2 - 2z = y^2$
2. $x^2 + 2x + y^2 = z^2 - 4z + 1$
3. $x^2 + y^2 - 2y = 3 - z^2$
4. $y^2 + 4y + x^2 - 2x + 5 = -z^2$
5. $x - y^2 = 3 - z^2$
6. $y - x^2 = 2 + z^2$

Arc Length

The position vector of a child going down a slide is given (in meters) by $\mathbf{r}(t) = \langle \sin t, \cos t, 5 - t \rangle$. How long is the slide? Where is the child after sliding 3 meters?

Return of Projectile Motion

A projectile is fired at angle α with initial velocity v_0. (1) If the downward acceleration due to gravity is $-g$, express the projectile's position and velocity as a function of time. (2) Describe and sketch the particle's trajectory. (3) What is the projectile's velocity at the time it hits the ground? (4) Find the point (in time and space) when the particle's velocity is at a minimum. Sketch the velocity and acceleration vectors at this point.

Uniform Circular Motion

- Find the curvature of a circle of radius r.
- A parcticle moves with position $\mathbf{r}(t)$, velocity $\mathbf{v}(t)$, and acceleration $\mathbf{a}(t)$. If $|\mathbf{r}(t)|$ is constant, show that $\mathbf{r} \cdot \mathbf{v} = 0$ at all points in time. If $|\mathbf{v}(t)|$ is constant, show that $\mathbf{v} \cdot \mathbf{a} = 0$ at all points in time.
- An object is traveling in a circle with constant speed. Find the relation between the acceleration and velocity of the particle and the radius of the circle.