13.1-13.2: Vector Functions
Friday, February 12

Warmup

Critique the given answer to the following problem:

1. Do the lines given by \( L_1 = \langle 1, 2, 3 \rangle + t\langle 2, -1, -2 \rangle \) and \( L_2 = \langle 3, 6, -1 \rangle + t\langle -1, 3, 0 \rangle \) intersect?

*Check whether the lines intersect by seeing whether there is a value of \( t \) such that the two points given by \( t \) are equal:*

\[
\langle 1, 2, 3 \rangle + t\langle 2, -1, -2 \rangle = \langle 3, 6, -1 \rangle + t\langle -1, 3, 0 \rangle
\]

\[
1 + 2t = 3 - t \\
2 - t = 6 + 3t \\
3 - 2t = -1.
\]

*The first equation gives \( t = 2/3 \), the second gives \( t = -1 \), and the third gives \( t = 2 \). There is no value of \( t \) that satisfies all three equations, and the lines therefore do not intersect.*

Sketch the curves given by the following equations:

1. \( z = y^2 - x \)
2. \( z^2 = x^2 \)
3. \( x^2 = z + y^2 + 1 \)
4. \( z = \sin t, x = \cos t, y = 1 \)

Space Curves

At what points does the curve \( r(t) = ti + (2t - t^2)k \) intersect the paraboloid \( z = x^2 + y^2 \)?
Derivatives and Integrals

A particle is moving along the curve $x = t, y = t^2 - t$.

1. Find the unit vector tangent to its path at $t = 2$.
2. Find its speed at $t = 2$.
3. At what points in time is the particle moving directly away from the origin?
4. When and where is the particle’s speed at a minimum?

Find a vector equation for the tangent line to the curve of intersection of the cylinders $x^2 + y^2 = 25$ and $y^2 + z^2 = 20$ at the point $(3, 4, 2)$.

The curves $r_1(t) = \langle t, t^2, t^3 \rangle$ and $r_2(t) = \langle \sin t \sin 2t, t \rangle$ intersect at the origin. Explain how to find their angle of intersection.

Bonus: Graph the curve with parametric equations $x = \cos t, y = \sin t \cos t, z = \sin^2 t$. It may help to graph the projections of the curve on the $x$-$y$, $x$-$z$, and $y$-$z$ planes.