13.1-13.2: Vector Functions

Friday, February 12

Warmup

Critique the given answer to the following problem:

1. Do the lines given by $L_1 = \langle 1, 2, 3 \rangle + t\langle 2, -1, -2 \rangle$ and $L_2 = \langle 3, 6, -1 \rangle + t\langle -1, 3, 0 \rangle$ intersect?

   Check whether the lines intersect by seeing whether there is a value of t such that the two points given by t are equal:

$$\begin{aligned} \langle 1, 2, 3 \rangle + t\langle 2, -1, -2 \rangle &= \langle 3, 6, -1 \rangle + t\langle -1, 3, 0 \rangle \\ 1 + 2t &= 3 - t \\ 2 - t &= 6 + 3t \\ 3 - 2t &= -1. \end{aligned}$$

   The first equation gives $t = 2/3$, the second gives $t = -1$, and the third gives $t = 2$. There is no value of t that satisfies all three equations, and the lines therefore do not intersect.

Sketch the curves given by the following equations:

1. $z = y^2 - x$
2. $z^2 = x^2$
3. $x^2 = z + y^2 + 1$
4. $z = \sin t$, $x = \cos t$, $y = 1$

Space Curves

At what points does the curve $\mathbf{r}(t) = t\mathbf{i} + (2t - t^2)\mathbf{k}$ intersect the paraboloid $z = x^2 + y^2$?

Derivatives and Integrals

A particle is moving along the curve $x = t, y = t^2 - t$.

1. Find the unit vector tangent to its path at $t = 2$.
2. Find its speed at $t = 2$.
3. At what points in time is the particle moving directly away from the origin?
4. When and where is the particle's speed at a minimum?

Find a vector equation for the tangent line to the curve of intersection of the cylinders $x^2 + y^2 = 25$ and $y^2 + z^2 = 20$ at the point $(3, 4, 2)$.

The curves $\mathbf{r}_1(t) = \langle t, t^2, t^3 \rangle$ and $\mathbf{r}_2(t) = \langle \sin t \sin 2t, t \rangle$ intersect at the origin. Explain how to find their angle of intersection.

Bonus: Graph the curve with parametric equations $x = \cos t, y = \sin t \cos t, z = \sin^2 t$. It may help to graph the projections of the curve on the x-y, x-z, and y-z planes.