12.4-5: Cross Product, Lines and Planes

Friday, February 5

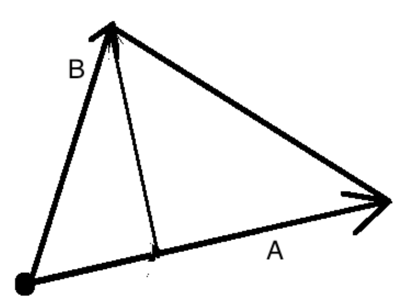
Warmup

1. Find the projection of the vector $\langle 1, 2\rangle$ onto $\langle 2, -1\rangle$. Make a sketch.
2. Find the projection of the vector $\langle 3, 2, 1\rangle$ onto $\langle 2, 0, 0\rangle$. Make a sketch.
3. One student is pushing a door open with a force of 50N at a 45° angle relative to the body of the door. Another is pushing it shut with a force of 40N perpendicular to the body of the door. A third is pushing with a force of 30N along the axis from the end of the door to its hinge. Which way will the door move? Make a sketch.
4. Find the determinant of the matrix
$$\begin{pmatrix} -1 & -1 & 1 \\ 1 & -2 & 0 \\ 0 & 3 & 3 \end{pmatrix}.$$
What does it tell you about the volume of a parallelepiped with one vertex at the origin and adjacent vertices at $(0, 3, 3), (1, -2, 0)$, and $(-1, -1, 1)$?

Matching

What do each of the following expressions have to say about the picture below? (Suppose the picture is embedded in three-dimensional space, so the cross product is well-defined.)

1. $a \times b$
2. $b \times a$
3. $(a \cdot b)/|a|$
4. $a(a \cdot b)/|a|^2$
5. $b - a(a \cdot b)/|a|^2$
6. $|a \times b|/|a|$
7. $|a \times b|/2$



Lines and Planes

1. If a plane is represented in the form $ax + by + cz = d$, what is the relation between $\langle a, b, c\rangle$ and the plane?

2. If the plane can also be written in the form $\{P_0 + t\mathbf{u} + s\mathbf{v} : s, t \in \mathbb{R}\}$, what is the relation between $\mathbf{u}, \mathbf{v}$, and $\langle a, b, c\rangle$?

3. How would you describe the set of points $\mathbf{x}$ such that $\mathbf{a} \cdot \mathbf{x} = 1$ for some nonzero vector $\mathbf{a}$ in $\mathbb{R}^2$? $\mathbb{R}^3$? $\mathbb{R}^4$??

Many Formulas

1. Given two parallel lines of the form $\mathbf{u_0} + t\mathbf{v}$ and $\mathbf{v_0} + t\mathbf{v}$, find a formula for the distance between the two lines. (Hint: it's the height of a triangle).

2. Find a formula for the distance between two planes in the form $\mathbf{n} \cdot x = k_1$ and $\mathbf{n} \cdot x = k_2$.

3. How can you use triple products to tell whether two lines of the form $\mathbf{u_0} + t\mathbf{u}$ and $\mathbf{v_0} + t\mathbf{v}$ are skew lines?

4. Come up with some questions of your own!