16.7-8: Surface Integrals, Stokes' Theorem

Friday, April 29

Generic Surface Integral

(16.7.23) Find the surface integral $\iint_S \mathbf{F} \cdot dS$ where $\mathbf{F}(x, y, z) = \langle xy, yz, zx \rangle$ and S is the part of the paraboloid $z = 4 - x^2 - y^2$ lying above the square $0 \leq x, y \leq 1$ and has upward orientation.

Special Surface Integral

(16.7.49) An electric charge at the origin generates an electric field given by $\mathbf{E}(r, \theta, \phi) = \frac{cr}{|r|^3}$, where c is a constant. Show that if S is the surface of a sphere centered at the origin then $\iint_S \mathbf{E} \cdot d\mathbf{S}$ does not depend on the radius of the sphere. What does this mean?

Stokes' Theorem

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$$

(16.8.7) Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y, z) = \langle x + y^2, y + z^2, z + x^2 \rangle$ and C is the (counterclockwise-oriented) boundary of the triangle with vertices $(1, 0, 0), (0, 1, 0), (0, 0, 1)$.

True or False?

1. If $\mathbf{F}$ is a vector field then $\nabla \cdot \mathbf{F}$ is a vector field.
2. If $\mathbf{F}$ is a vector field then $\nabla \times \mathbf{F}$ is a vector field.
3. I f has continuous partial derivatives on $\mathbb{R}^3$ then $\nabla \cdot (\nabla \times f) = 0$.
4. If f has continuous partial derivatives on $\mathbb{R}^3$ and C is any circle then $\int_C \nabla f \cdot d\mathbf{r} = 0$.
5. If $\mathbf{F} = \langle P, Q \rangle$ and $P_y = Q_x$ in an open region D then $\mathbf{F}$ is conservative.
6. If $\mathbf{F}$ and $\mathbf{G}$ are vector fields and $\nabla \times \mathbf{F} = \nabla \times \mathbf{G}$ then $\mathbf{F} = \mathbf{G}$.
7. The work done by a conservative force field in moving a particle around a closed path is zero.
8. There is a vector field $\mathbf{F}$ such that $\nabla \times \mathbf{F}\langle x, y, z \rangle$.