16.3-5: Green's Theorem, Curl

Friday, April 22

Work

(16.3.23) Find the work done by the force field $\mathbf{F}(x, y) = \langle 2y^{3/2}, 3x\sqrt{y} \rangle$ in moving an object from $(1, 1)$ to $(2, 4)$.

Green's Theorem

(16.4.21) If C is the line segment from (x_1, y_1) to (x_2, y_2), show that

$$\int_C x\,dy - y\,dx = x_1y_2 - x_2y_1.$$

Use this to find a formula for the area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$.

Curl

Use the curl operator to determine whether the vector field $\mathbf{F}(x, y, z) = \langle y^2 z^3, 2xyz^3, 3xy^2 z^2 \rangle$ is conservative.

Divergence

A charged particle at the origin generates the electric field $\mathbf{E}(x, y) = \langle x/(x^2 + y^2)^{3/2}, y/(x^2 + y^2)^{3/2} \rangle$.

1. Find $\nabla \cdot \mathbf{E}$.
2. Find curves C_1 and C_2 such that $\oint_{C_1} \mathbf{E} \cdot \mathbf{n}\, ds = 0$ and $\oint_{C_2} \mathbf{E} \cdot \mathbf{n}\, ds \neq 0$.

True/False

1. If $\mathbf{F}$ is conservative then $\nabla \times \mathbf{F} = 0$.
2. If $\mathbf{F}$ is conservative then $\nabla \cdot \mathbf{F} = 0$.
3. If $\nabla \times \mathbf{F} = 0$ then $\mathbf{F}$ is conservative.
4. Green's Theorem is just the Divergence Theorem in two dimensions.
5. curl(div($\mathbf{F}$)) is not a meaningful expression.