16.2-3: Line Integrals

Friday, April 15

Recap: Change of Coordinates

(15.9.15) Evaluate the integral $\iint_R (x-3y)\,dA$ where R is the triangular region with vertices $(0,0), (2,1), (1,2)$ given the transformation $x = 2u + v,\ y = u + 2v$.

Line Integrals

Find the work done by the force field $\mathbf{F}(x,y) = \langle x - y^2, y - x^2 \rangle$ on a praticle that moves along the line segment from $(0,0)$ to $(2,1)$.

A student swings a ball of mass m on a string of radius r in a vertical circle. Use a line integral to calculate the work that gravity does on the ball (given constant downward force mg)...

1. as the ball goes from the top of its arc to the bottom.
2. over one complete revolution.

True or False?

1. The integral $\int_{\phi=0}^{\pi/2}\int_{\theta=0}^{\pi/2}\int_{\rho=0}^{1}\rho^2\sin\theta\,d\rho\,d\theta\,d\phi$ gives the volume of 1/4 of a sphere.
2. $\int_{r=-1}^{1}\int_{\theta=0}^{1}e^{r^2+\theta^2}\,d\theta\,dr=\left[\int_{r=-1}^{1}e^{r^2}\,dr\right]\left[\int_{\theta=0}^{1}e^{\theta^2}\,d\theta\right]$
3. If C is a closed curve then $\int_C f\,ds=0$ for any function f.
4. If $\int_C f\,ds=0$ then C is a closed curve.
5. If the work done by a force $\mathbf{F}$ on an object moving along a curve is W, then if the object moves along the curve in the opposite direction the work done by $\mathbf{F}$ will be $-W$.
6. If a particle moves along a curve C, the total work done by a force $\mathbf{F}$ on the object is independent of how quickly the particle moves.
7. If a force points only in the x direction then the work done by the force on a particle depends only on the particle's starting and ending x-positions.