Midterm 2: Review I

Friday, April 1

Gradients

What is a gradient, and what is it good for? Give sketches and examples including but not limited to contour plots, limits, linear approximation, the Chain Rule, and optimization problems.

Second Derivative Test

Given a function $f : \mathbb{R}^2 \to \mathbb{R}$, what is the Second Derivative test? Give all possible outcomes of the test and say what they tell you, providing sketches.

True or False?

1. If ∇f exists everywhere then f is continuous everywhere.
2. If $f_x = f_y = 0$ at a point (x, y) then f is differentiable at (x, y).
3. If f is differentiable along every straight line going through a point (x, y) then f is differentiable at (x, y).
4. For any x, $f(x - \nabla f(x)) \leq f(x)$.
5. If $f(x, y) = 1$ then $\iint_D f(x, y)\, dA$ is equal to the area of the domain D.
6. For any $a, b \in \mathbb{R}$ and continuous function f, $\int_{x=0}^{a} \int_{y=0}^{b} f(x, y)\, dy\, dx = \int_{y=0}^{b} \int_{x=0}^{a} f(x, y)\, dx\, dy$.
7. If $f(x, y) = g(x)h(y)$, then $\iint_D f(x, y)\, dA = \left(\iint_D g(x)\, dA\right) \left(\iint_D h(y)\, dA\right)$.
8. If $f_{xx} > 0$ and $f_{yy} > 0$ at a point (x, y) then the point (x, y) is a local minimum of the function f.
9. If (x, y) is a local minimum of a function f then f is differentiable at (x, y) and $\nabla f(x, y) = 0$.
10. If $\nabla f(x, y) = 0$ then (x, y) is a local minimum or maximum of f.
11. If $f_{xx} > 0$ and $f_{yy} < 0$ at a point (x, y) then (x, y) is a saddle point of f.
12. If ∇f is never zero then the minimum and maximum of f on a closed and bounded domain D must occur on the boundary.
13. If f has a critical point in the interior of a closed and bounded domain D then the minimum and maximum of f on D occur in the interior of D.
14. If x is a minimum of f given the constraints $g(x) = h(x) = 0$ then $\nabla f(x) = \lambda \nabla g(x)$ and $\nabla f(x) = \mu \nabla h(x)$ for some scalars λ and μ.