Quiz 6; Wednesday, March 2
MATH 53 with Professor Stankova
Section 116; 3-4
GSI: Eric Hallman

Student name:

You have 10 minutes to complete the quiz. Calculators are not permitted, and remember to show your calculations and explain your reasoning in order to receive full credit.

1. Consider the function $f(x,y) = \dfrac{x^2y}{x^2+y^4}$. Find $\lim\limits_{(x,y)\to(2,1)} f(x,y)$.

   The function is continuous at $(2,1)$, so $\lim\limits_{(x,y)\to(2,1)} f(x,y) = f(2,1) = \dfrac{2^2\cdot 1}{2^2+1^4} = 4/5$.

2. Make a conjecture about $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ by finding the limit along at least one line and at least one non-linear curve.

   Along the line $x=0$: $\lim\limits_{y\to 0} f(0,y) = \lim\limits_{y\to 0} \dfrac{0}{y^4} = 0$.

   Along the line $y=0$: $\lim\limits_{x\to 0} f(x,0) = \lim\limits_{x\to 0} \dfrac{0}{x^2} = 0$.

   Along the line $y=x$: $\lim\limits_{y\to 0} f(y,y) = \lim\limits_{y\to 0} \dfrac{y^3}{y^2+y^4} = \lim \dfrac{y}{1+y^2} = 0$.

   Along the curve $x=y^2$: $\lim\limits_{y\to 0} f(y^2,y) = \lim\limits_{y\to 0} \dfrac{y^5}{2y^4} = \lim\limits_{y\to 0} \dfrac{y}{2} = 0$.

   We conjecture that the limit exists and is equal to zero (not that this does not prove that the limit exists!)

   Simplest way to prove that the limit exists: $0 \le \frac{x^2}{x^2+y^4} \le 1$ for any x and y such that $(x,y)\neq(0,0)$, so $\left|\frac{x^2y}{x^2+y^4}\right| \le |y|$ and therefore $-|y| \le \frac{x^2y}{x^2+y^4} \le |y|$ for any $(x,y)\neq(0,0)$. By the Squeeze Theorem (since $\lim_{x,y\to(0,0)} |y| = 0$), the limit is zero.