Quiz 3; Wednesday, February 10
MATH 53 with Professor Stankova
Section 116; 3-4
GSI: Eric Hallman

Student name:

You have 10 minutes to complete the quiz. Calculators are not permitted, and remember to show your calculations and explain your reasoning in order to receive full credit.

1. Find the minimum distance between the line that goes through the points $A(0,1,3), B(1,2,5)$ and the line that goes through the points $C(1,-2,1)$ and $D(0,0,0)$.

   The first step is to find the vectors that determine these lines: Line 1 is $\langle 0,1,3\rangle + t\langle 1,1,2\rangle$ in vector form, and Line 2 is $t\langle 1,-2,1\rangle$.

   Then find a vector perpendicular to both of these vectors, since the line segment determining the shortest path will be perpendicular to both lines. The simplest way to do this is to use the cross product:

$$\begin{aligned}\langle 1,1,2\rangle \times \langle 1,-2,1\rangle &= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & 1 & 2\\ 1 & -2 & 1\end{vmatrix}\\ &= 5i + j - 3k\\ &= \langle 5,1,-3\rangle.\end{aligned}$$

   Then take the vector from any point on line 2 to any point on line 1, and its dot product with the normal vector will be the same. So use $\langle 0,0,0\rangle$ and $\langle 0,1,3\rangle$, and $\langle 0,1,3\rangle \cdot \langle 5,1,-3\rangle = -8$. The distance between the two lines is therefore $8/|\langle 5,1,-3\rangle| = 8/\sqrt{35}$.