Quiz 11; Wednesday, April 13
MATH 53 with Professor Stankova
Section 116; 3-4
GSI: Eric Hallman

Student name:

You have 10 minutes to complete the quiz. Calculators are not permitted, and remember to show your calculations and explain your reasoning in order to receive full credit.

1. Evaluate the following integral by making the change of coordinates $u = x - 2y, v = 3x - y$:

$$\iint_R \frac{x-2y}{3x-y}\,dA$$

where R is the parallelogram enclosed by the lines $x - 2y = 0, x - 2y = 4, 3x - y = 1, 3x - y = 8$.

First we need to find the Jacobian of the transformation:

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 1 & -2 \\ 3 & -1 \end{vmatrix} = 5,$$

or express x and y in terms of u and v to get $y = (v - 3u)/5$, $x = (2v - u)/5$, and then

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} -1/5 & 2/5 \\ -3/5 & 1/5 \end{vmatrix} = 1/5,$$

so either way we get the substitution $dx\,dy = \frac{1}{5}\,du\,dv$.

Next, the domain R get transformed to a rectangle with the constraints $0 \le u \le 4, 1 \le v \le 8$, and so the integral becomes

$$\begin{aligned} \iint_R \frac{x-2y}{3x-y}\,dA &= \int_{u=0}^{4}\int_{v=1}^{8} \frac{u}{v}\frac{1}{5}\,du\,dv \\ &= \frac{1}{5}[\frac{1}{2}u^2]_0^4[\ln v]_1^8 \\ &= \frac{1}{5}(8)(\ln 8) \\ &= \frac{8\ln 8}{5}. \end{aligned}$$