

**Quiz 10;** Wednesday, April 6  
**MATH 53** with Professor Stankova  
**Section 116;** 3-4  
**GSI:** Eric Hallman

**Student name:**

You have 10 minutes to complete the quiz. Calculators are not permitted, and remember to show your calculations and explain your reasoning in order to receive full credit.

1. Use a double integral in polar coordinates to find the volume of the solid below the paraboloid  $z = 18 - 2x^2 - 2y^2$  and above the xy-plane.

We need to find the bounds of the domain, so look for where the paraboloid intersects the xy-plane (at  $z=0$ ): if  $0 = 18 - 2x^2 - 2y^2$  then  $x^2 + y^2 = 9$ , so in polar coordinates we should integrate up to  $r = 3$ :

$$\begin{aligned}\int_{\theta=0}^{2\pi} \int_{r=0}^3 (18 - 2r^2)r \, dr \, d\theta &= 2\pi \int_{r=0}^3 18r - 2r^3 \, dr \\ &= 2\pi \left[ 9r^2 - \frac{1}{2}r^4 \right]_{r=0}^3 \\ &= 2\pi(81 - 81/2) \\ &= 81\pi.\end{aligned}$$