Quiz 8; Wednesday, March 16
MATH 53 with Professor Stankova
Section 109; 11-12
GSI: Eric Hallman

Student name:

You have 10 minutes to complete the quiz. Calculators are not permitted, and remember to show your calculations and explain your reasoning in order to receive full credit.

1. Find the maximum and minimum values of the function $f(x, y, z) = x + 2y + 2z$ on the domain $x^2 + y^2 + z^2 = 1$.

   ANSWER: $\nabla f(x, y, z) = \langle 1, 2, 2 \rangle$, which is never zero. Therefore the maximum and minimum values occur on the boundary, which we can find using Lagrange multipliers. If $g(x, y, z) = x^2 + y^2 + z^2$ then $\nabla g(x, y, z) = 2\langle x, y, z \rangle$ and we get the equations

$$
\begin{aligned}
1 &= \lambda x \\
2 &= \lambda y \\
2 &= \lambda z,
\end{aligned}
$$

   which means that $2x = y = z$. Substituting this into the constraint $x^2 + y^2 + z^2 = 1$ then gives that $9x^2 = 1$, so $x = \pm 1/3$. The two critical points are therefore $(x, y, z) = \pm(1/3, 2/3, 2/3)$. f hits a maximum at the point with positive coordinates and a minimum at the point with negative coordinates.