You have 10 minutes to complete the quiz. Calculators are not permitted, and remember to show your calculations and explain your reasoning in order to receive full credit.

1. Find the area that lies inside the curve $r = 4 \sin \theta$ but outside the curve $r = 2$.

First find the points of intersection: $4 \sin \theta = 2$ when $\theta = \pi/6, 5\pi/6$. Then take the appropriate integral (since the inner curve is a sector of a circle we can find its area without taking an integral):

$$A = \int_{\theta = \pi/6}^{5\pi/6} \frac{1}{2} r^2 \, d\theta - \frac{4\pi}{3}$$

$$= 8 \int_{\theta = \pi/6}^{5\pi/6} \sin^2 \theta \, d\theta - \frac{4\pi}{3}$$

$$= 4 \int_{\theta = \pi/6}^{5\pi/6} (1 - \cos 2\theta) \, d\theta - \frac{4\pi}{3}$$

$$= 4 \left( \theta - \frac{\sin 2\theta}{2} \right)_{\pi/6}^{5\pi/6} - \frac{4\pi}{3}$$

$$= 4\pi/3 + 2\sqrt{3}.$$