Quiz 11; Wednesday, April 13
MATH 53 with Professor Stankova
Section 109; 11-12
GSI: Eric Hallman

Student name:

You have 10 minutes to complete the quiz. Calculators are not permitted, and remember to show your calculations and explain your reasoning in order to receive full credit.

1. Evaluate $\iiint_E xe^{x^2+y^2+z^2}\,dV$ where E is the portion of the unit ball $x^2+y^2+z^2 \leq 1$ lying in the first octant $(x, y, z \geq 0)$.

   The unit ball constraint gives the bounds $0 \leq \rho \leq 1$ and constraining the domain to the first octant is equivalent to the bounds $0 \leq \theta, \phi \leq \pi/2$. As $x^2+y^2+z^2 = \rho^2$ and $x = \rho\sin\phi\cos\theta$, the integral becomes

$$\begin{aligned}
\iiint_E xe^{x^2+y^2+z^2}\,dV &= \int_{\rho=0}^{1}\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{\pi/2}(\rho\sin\phi\cos\theta)e^{\rho^2}\rho^2\sin\phi\,d\phi\,d\theta\,d\rho \\
&= \left(\int_{\rho=0}^{1}\rho^3e^{\rho^2}\,d\rho\right)\left(\int_{\theta=0}^{\pi/2}\cos\theta\,d\theta\right)\left(\int_{\phi=0}^{\pi/2}\sin^2\phi\,d\phi\right) \\
&= ([\frac{1}{2}\rho^2e^{\rho^2} - \frac{1}{2}e^{\rho^2}]_0^1)([\sin\theta]_0^{\pi/2})([\theta/2 - \sin(2\theta)/4]_0^{\pi/2}) \\
&= (1/2)(1)(\pi/4) \\
&= \pi/8.
\end{aligned}$$