Quiz 10; Wednesday, April 6
MATH 53 with Professor Stankova
Section 109; 11-12
GSI: Eric Hallman

Student name:

You have 10 minutes to complete the quiz. Calculators are not permitted, and remember to show your calculations and explain your reasoning in order to receive full credit.

1. Use a double integral in polar coordinates to find the volume of a sphere with radius a.

   Integrate the function $z = \sqrt{a^2 - r^2}$, making sure to multiply by 2 to count the top and bottom halves of the sphere:

$$\begin{aligned} 2\int_{r=0}^{a}\int_{\theta=0}^{2\pi} r\sqrt{a^2-r^2}\,dr\,d\theta &= 4\pi\int_{r=0}^{a} r\sqrt{a^2-r^2}\,dr \\ &= 4\pi[-\frac{1}{3}(a^2-r^2)^{3/2}]_{r=0}^{a} \\ &= \frac{4}{3}\pi a^3. \end{aligned}$$