Quiz 10: Wednesday, April 6
MATH 53 with Professor Stankova
Section 109: 11-12
GSI: Eric Hallman

Student name:

You have 10 minutes to complete the quiz. Calculators are not permitted, and remember to show your calculations and explain your reasoning in order to receive full credit.

1. Use a double integral in polar coordinates to find the volume of a sphere with radius $a$.

   Integrate the function $z = \sqrt{a^2 - r^2}$, making sure to multiply by 2 to count the top and bottom halves of the sphere:

   $$
   2 \int_{r=0}^{a} \int_{\theta=0}^{2\pi} r \sqrt{a^2 - r^2} \, dr \, d\theta = 4\pi \int_{r=0}^{a} r \sqrt{a^2 - r^2} \, dr
   $$

   $$
   = 4\pi \left[ -\frac{1}{3} (a^2 - r^2)^{3/2} \right]_{r=0}^{a}
   $$

   $$
   = \frac{4}{3} \pi a^3.
   $$