True or False Problems

1. If $a_n, b_n > 0$ and $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} b_n$ diverges then $\sum_{n=1}^{\infty} a_n b_n$ diverges.
   FALSE: $a_n = 1/n^2$, $b_n = 1$.
2. If $a_n, b_n > 0$ and $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} b_n$ diverges then $\sum_{n=1}^{\infty} a_n + b_n$ diverges.
   TRUE
3. If $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} b_n$ converges then $\sum_{n=1}^{\infty} a_n + b_n$ converges.
   TRUE
4. If $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} a_n^2$ converges.
   FALSE: $a_n = (-1)^n/\sqrt{n}$.
5. If $a_n > 0$ and $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} a_n^2$ converges.
   TRUE (by Limit Comparison Test)
6. If $a_n, b_n > 0$ and $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} b_n$ converges then $\sum_{n=1}^{\infty} a_n/b_n$ converges.
   FALSE: $a_n = b_n = 1/n^2$.
7. If $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} n a_n$ converges.
   FALSE: $a_n = 1/n^2$.
8. If $\{a_n\}$is increasing and bounded above then it is convergent.
   TRUE
9. If $\{a_n\}$is decreasing and positive then it is convergent.
   TRUE
10. If $\{a_n\}$is increasing and positive then it is convergent.
    FALSE: $a_n = n$
11. If $\{a_n\}$is monotonic and bounded below then it is convergent.
    FALSE: $a_n = n$
12. If $\{a_n\}$is convergent and bounded then it is monotonic.
    FALSE: $a_n = \sin(n)/n$
13. If $a_n > 0$ and $\lim_{n\to\infty} a_n = 0$ then $\{a_n\}$ is monotonically decreasing.
    FALSE: $a_n = \sin^2(n)/n$
14. If $\sum_{n=1}^{\infty} a_n$ converges then it converges absolutely.
    FALSE: $a_n = (-1)^n/n$
15. If $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} a_n$ converges.
    TRUE
16. If $\sum_{n=1}^{\infty} a_n$ converges absolutely then $\sum_{n=1}^{\infty} \sin(a_n) a_n$ converges.
    TRUE
17. If $\sum_{n=1}^{\infty} a_n$ converges conditionally then $\sum_{n=1}^{\infty} a_n/\sqrt{n}$ converges absolutely.
    FALSE: $a_n = (-1)^n/\sqrt{n}$

18. If $\sum_{n=1}^{\infty} a_n$ converges conditionally and $\{b_n\}$ is bounded then $\sum_{n=1}^{\infty} a_n b_n$ converges.
    FALSE: $a_n = (-1)^n/n, b_n = (-1)^n$

19. If $\sum_{n=1}^{\infty} a_n$ converges absolutely and $\{b_n\}$ is bounded then $\sum_{n=1}^{\infty} a_n b_n$ converges.
    TRUE

20. If the radius of convergence of $\sum_{n=1}^{\infty} a_n x^n$ is 3 then $\sum_{n=1}^{\infty} a_n$ is divergent.
    FALSE: $a_n = (1/3)^n$

21. If $\sum_{n=1}^{\infty} a_n$ is divergent then the radius of convergence of $\sum_{n=1}^{\infty} a_n x^n$ is $1/3$.
    FALSE: $a_n = 1$

22. If $c \in \mathbb{R}$ is some constant then the radius of convergence of $\sum_{n=1}^{\infty} c a_n x^n$ is the same as the radius of convergence of $\sum_{n=1}^{\infty} a_n x^n$.
    TRUE

23. If $P(x)$ is some polynomial then $\sum_{n=1}^{\infty} P(n) x^n$ has radius of convergence 1.
    TRUE