Trigonometric Substitutions

Wednesday, January 28

Speed Round

Simplify the following expressions:

1. $\tan^2(x) + 1$
2. $\sin^2(x) + \cos^2(x)$
3. $\sec^2(x) - 1$
4. $\sqrt{1 - \sin^2(x)}$
5. $\sqrt{1 + \tan^2(x)}$
6. $\sqrt{a^2 - a^2\cos^2(x)}$
7. $\sqrt{(a\sec(x))^2 - a^2}$
8. $1 - \cos^2(x)$

Decide which substitution would be most appropriate for evaluating each of the following integrals. Note: it will not always be a trig substitution.

1. $\int x\sqrt{1+x^2}\,dx$
2. $\int \frac{1}{\sqrt{1-x^2}}\,dx$
3. $\int \frac{1}{(x^2+1)^2}\,dx$
4. $\int \frac{1}{\sqrt{x^2-4}}\,dx$
5. $\int \frac{x}{\sqrt{1+x^2}}\,dx$
6. $\int \frac{x^2}{\sqrt{4x^2-9}}\,dx$
7. $\int \frac{\sqrt{x^2-1}}{x}\,dx$
8. $\int \frac{\sqrt{1+x^2}}{x}\,dx$
9. $\int x^3\sqrt{1-x^2}\,dx$
10. $\int \frac{1}{\sqrt{9-25x^2}}\,dx$
11. $\int \frac{x}{x^2-9}\,dx$
12. $\int \frac{x^2}{\sqrt{x^2-9}}\,dx$

Theorem: $\bigcirc = \pi r^2$

Prove (finally!) that the area of a circle of radius r is πr^2.

More Trig Identities

Simplify the following expressions:

1. $\sin(\arccos x)$
2. $\cos(\arcsin x)$
3. $\sec(\arctan x)$
4. $\cos(\arctan x)$
5. $\tan(\arcsin x)$
6. $\tan(\sec^{-1} x)$

1. If $t = \sec\theta$, then what is $\tan\theta$ in terms of t?
2. If $t = \sin\theta$, then what is $\cos\theta$ in terms of t?
3. If $t = \tan\theta$, then what is $\sec\theta$ in terms of t?

Completing the square

Put each of the following quadratics in the form $(Ax + B)^2 + C$, then decide which trig substitution would be appropriate:

1. $\sqrt{x^2 + 4x + 8}$
2. $\sqrt{-x^2 - 6x}$
3. $\sqrt{x^2 - 2x + 10}$
4. $\sqrt{-4x^2 - 12x - 5}$
5. $\sqrt{9x^2 - 6x - 3}$
6. $\sqrt{4x^2 - 4x + 2}$
7. $\sqrt{25x^2 + 10x + 10}$
8. $\sqrt{-16x^2 + 24x - 5}$

Bonus: Prove the quadratic formula.

Weierstrass substitution

$$t = \tan(x/2);\ \sin(x) = \frac{2t}{1+t^2};\ \cos(x) = \frac{1-t^2}{1+t^2};\ dx = \frac{2}{1+t^2}dt$$

Integrate:

1. $\int \frac{1}{\sin x}\,dx$
2. $\int \frac{1}{\cos x}\,dx$
3. $\int \frac{1}{1+\sin x}\,dx$
4. $\int \frac{1}{1+\cos x}\,dx$