

Trigonometric Integrals

Friday, January 23

Review

Compute the following integrals using integration by parts. It might be helpful to make a substitution.

1. $\int_1^{e^2} \sqrt{x} \ln(x) dx$

2. $\int_0^1 x\sqrt{1+x} dx$

Discuss: does the best strategy for solving each of the following integrals use substitution, integration by parts, both, or neither?

1. $\int x \ln(x) dx$

4. $\int 1/x dx$

7. $\int x\sqrt{1+x} dx$

2. $\int \frac{\ln(x)}{x} dx$

5. $\int \ln(x) dx$

8. $\int x\sqrt{x} dx$

3. $\int \frac{1}{x \ln(x)} dx$

6. $\int \cos(x)e^{\sin(x)} dx$

9. $\int \sin(x) \cos(x)e^{\sin(x)} dx$

Trig Formulas to Memorize:

1. $\sin^2(x) + \cos^2(x) = 1$

5. $\int \sin(x) dx = -\cos(x) + C$

2. $\sin(2x) = 2 \sin(x) \cos(x)$

6. $\int \cos(x) dx = \sin(x) + C$

3. $\cos(2x) = \cos^2(x) - \sin^2(x)$

7. $\int \sec^2(x) dx = \tan(x) + C$

4. $\tan^2(x) + 1 = \sec^2(x)$

8. $\int \sec(x) \tan(x) dx = \sec(x) + C$

Also Good to Know:

1. $\sin(a \pm b) = \sin(a) \cos(b) \pm \cos(a) \sin(b)$

4. $\cos(2x) = 1 - 2 \sin^2(x)$

2. $\cos(a \pm b) = \cos(a) \cos(b) \mp \sin(a) \sin(b)$

5. $\sin^2(x) = (1 - \cos(2x))/2$

3. $\cos(2x) = 2 \cos^2(x) - 1$

6. $\cos^2(x) = (1 + \cos(2x))/2$

Formulas to Write on a Cheat Sheet:

Everything else.

Speed Round

1. $\int \cos(x) dx$

6. $\int \sec^2(x) dx$

11. $\sqrt{1 - \sin^2(x)}$

2. $\int \sin(x) dx$

7. $(1 + \cos(x))(1 - \cos(x))$

12. $\frac{d}{dx} \tan(x)$

3. $\sin^2(x) + \cos^2(x)$

8. $\cos^4(x) - \sin^4(x)$

13. $\frac{d}{dx} \sec(x)$

4. $\sqrt{1 - \cos^2(x)}$

9. $(1 - x^2)/(1 - x)$

14. $\sec^2(x) - 1$

5. $(a + b)(a - b)$

10. $\cos^2(x)/(1 - \sin(x))$

15. $\cos(2x) + 1$

Identities

Prove the following trig identities using only $\cos^2(x) + \sin^2(x) = 1$ and sine and cosine addition formulas:

1. $\tan^2(x) + 1 = \sec^2(x)$
2. $\sin^2(x) = (1 - \cos(2x))/2$
3. $\cos^2(x) = (1 + \cos(2x))/2$
4. $\sin(a) \sin(b) = \frac{1}{2}[\cos(a - b) - \cos(a + b)]$

Integrals

Evaluate the following integrals:

1. $\int \sin^2(\sqrt{x})/\sqrt{x} dx$
2. $\int \sqrt{1 + \cos(2x)} dx$
3. $\int \frac{1}{1 + \sin(x)} dx$
4. $\int \tan(x) dx$
5. $\int \tan^2(x) dx$
6. $\int \tan^3(x) dx$
7. $\int \sqrt{1 - \cos(4x)} dx$
8. $\int \frac{1}{\cos(x) - 1} dx$

Bonus

1. Show that $\frac{1}{2} \ln \left(\frac{1 + \sin(x)}{1 - \sin(x)} \right) + C = \ln(\sec(x) + \tan(x)) + C$.
2. Evaluate $\int_0^{2\pi} \sin(3x) \sin(5x) \sin(7x) dx$