11.10: Taylor Series

Wednesday, March 18

Recap

A certain power series centered at $x = 2$ converges at -2 and diverges at 7. Decide whether it converges or diverges at each of the following points, or whether you do not have enough information to tell.

1. $x = -4$
2. $x = -3$
3. $x = 2$
4. $x = 5$
5. $x = 6$
6. $x = 8$

Power Series

1. $e^x =$
2. $\sin x =$
3. $\cos x =$
4. $\frac{1}{1-x} =$
5. $\ln(1 + x) =$
6. $\arctan(x) =$

Power Series Arithmetic

1. $\frac{1}{1+2x} =$
2. $e^{2x} + \sin(x) =$
3. $\sin x + 2\cos 2x =$
4. $(\sin x)^2 =$
5. Show that $\frac{d}{dx} \sin x = \cos x$.
6. Derive the Taylor series for $\arctan x$ at $x = 0$ by integrating $\frac{1}{1+x^2}$.
7. Show that $\sin 2x = 2 \sin x \cos x$, at least up to the x^3 term in their series expansions.

Taylor Series

Find the Taylor series expansions for the given functions around the given points.

1. $\sqrt{3 + x}$ around $x = 0$
2. $\sqrt{x}$ around $x = 3$
3. $\sin(x)$ around $x = \pi/2$
4. e^{2x} around $x = 1$

Find the Taylor series expansions for the function $f(x) = x^3 - 3x$ at $x = 0$, $x = 1$, and $x = 2$. Sketch the linear and quadratic approximations at each of those points below:

1. Are the three full Taylor series expansions the same? If not, how do they differ?

2. What do the coefficients of the Taylor series expansions tell you about the behavior of the function (e.g. slope, concavity) at each of the three points?

3. Compare the Taylor series expansions for $\sin x$ around $x = 0$ and $x = 2\pi$. How are these similar? How do they differ?