Speed Review

Monday, February 9

Basic Identities

1. $\sin^2 x + \cos^2 x = 1$
2. $\tan^2 x + 1 = \sec^2 x$
3. $\tan x \cos x = \sin x$
4. $\sec^2 x - 1 = \tan^2 x$
5. $\sqrt{1 - \sin^2 x} = \cos x$
6. $\sqrt{1 + \tan^2 x} = \sec x$
7. $(1 + \sin x)(1 - \sin x) = \cos^2 x$
8. $\int \sec^2 x \, dx = \tan x$
9. $\int \sec x \tan x \, dx = \sec x$
10. $\int 1/\cos^2 x \, dx = \tan x$
11. $\int \tan x \cos x \, dx = -\cos x$
12. $\sin 2x = 2 \sin x \cos x$
13. $\cos 2x = \cos^2 x - \sin^2 x$
14. $1 - \cos 2x = 2\sin^2 x$
15. $1 + \cos 2x = 2\cos^2 x$

Strategies for Integration

If integration by parts is necessary, circle your choice for u. If the first step is instead to make a substitution, write your choice of substitution.

1. $\displaystyle\int x^3\sqrt{1-x^2}\,dx$: substitute $y = x^2$ or $x = \sin\theta$
2. $\displaystyle\int xe^{x^2}\,dx$: substitute $y = x^2$
3. $\displaystyle\int \arcsin(x)\,dx$: IBP $u = \arcsin x$
4. $\displaystyle\int \frac{1}{x^2\sqrt{x^2-1}}\,dx$: sub $x = \sec\theta$
5. $\displaystyle\int \frac{1}{x\ln x}\,dx$: sub $y = \ln x$
6. $\displaystyle\int \frac{x^5}{x^2+1}\,dx$: sub $y = x^2$ or $x = \tan\theta$
7. $\displaystyle\int x\ln(x)\,dx$: IBP $u = \ln x$
8. $\displaystyle\int e^{\sqrt{x}}\,dx$: sub $y = \sqrt{x}$
9. $\displaystyle\int x\sin(x)\,dx$: IBP $u = x$
10. $\displaystyle\int x\sqrt{1+x^2}\,dx$: sub $y = 1 + x^2$ (or $x = \tan\theta$)
11. $\displaystyle\int \frac{x^2}{\sqrt{9-x^2}}\,dx$: sub $x = 3\sin\theta$
12. $\displaystyle\int xe^x\,dx$: IBP $u = x$
13. $\displaystyle\int \cos(x)\ln(\sin x)\,dx$: sub $y = \sin x$

14. $\displaystyle\int \frac{x}{\sqrt{4+x^2}}\,dx$: sub $u = 4 + x^2$ (or $x = 2\tan\theta$)

15. $\displaystyle\int \frac{\sqrt{x^2-9}}{x^3}\,dx$: sub $x = \sec\theta$

16. $\displaystyle\int x^3\cos(x^2)\,dx$: sub $y = x^2$

17. $\displaystyle\int x\arctan(x)\,dx$: IBP $u = \arctan x$

18. $\displaystyle\int (x^2+2x)e^x\,dx$: IBP $u = x^2 = 2x$ (or split the integral first)

Partial Fractions

Write out the partial fraction form of each of the following functions. Specify whether long division is necessary at the start.

1. $\left(\frac{x}{x+1}\right)^2 = 1 + \frac{A}{x+1} + \frac{B}{(x+1)^2}$ (long division first)
2. $\frac{x^3}{(x+1)(x+4)} = Ax + B + \frac{C}{x+1} + \frac{D}{x+4}$ (long division first)
3. $\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$
4. $\frac{x+3}{(x+1)^2(x^2+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+1} + \frac{Ex+F}{(x^2+1)^2}$
5. $\frac{x^4+5}{(x-1)^2(x+2)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} + \frac{D}{(x+2)^2} + \frac{E}{(x+2)^2}$

Numerical Methods

1. If we want to estimate $\int_0^5 17x + 8\,dx$ using the Trapezoidal Rule, how many intervals do we need to guarantee that our error is less than 0.01?

   Just one interval will give the correct answer.

2. If we want to estimate $\int_0^4 4x^2 - 3x + 2\,dx$ using Simpson's Rule, how many intervals do we need to guarantee that our error is less than 0.01?

   Just two intervals (three points, to make a single parabola) will give the correct answer.

Convergent or Divergent?

1. $\int_0^1 \frac{1}{x}\,dx$: DIVERGENT
2. $\int_0^1 \frac{1}{x^2}\,dx$: DIVERGENT
3. $\int_0^1 \frac{1}{\sqrt{x}}\,dx$: CONVERGENT
4. $\int_1^\infty \frac{1}{x}\,dx$: DIVERGENT
5. $\int_1^\infty \frac{1}{x^2}\,dx$: CONVERGENT
6. $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$: DIVERGENT
7. $\int_0^1 \frac{1}{(x-1)^2}\,dx$: DIVERGENT
8. $\int_0^1 \frac{\sin x}{x}\,dx$: CONVERGENT
9. $\int_0^2 \frac{x+2}{(x-1)^{2/3}}\,dx$: CONVERGENT
10. $\int_2^\infty \frac{1}{\ln x}\,dx$: DIVERGENT
11. $\int_0^1 \ln(x)\,dx$: CONVERGENT
12. $\int_0^1 \frac{x}{\tan x}\,dx$: CONVERGENT