Sequences

Wednesday, Febrary 18

Review

1. $\ln(a \cdot b) =$
2. $\ln(x^a) =$
3. $x^y = e^?$
4. $5! =$
5. $e^x = 1 + x + \dots$
6. $(a+b)(a-b) =$
7. $\ln(n+1) - \ln(n) = \ln(?)$
8. $\sqrt{n^2+n} - n = ?$
9. $\lim_{x \to 0} \frac{\sin x}{x}$

Sequences

Write the first four or five terms (starting from $n = 0$) of each of the following sequences:

1. $a_n = 6 \cdot \left(\frac{2}{3}\right)^n$
2. $a_n = (-1)^n/(2n+1)$
3. $a_n = n! - 3^n$

Find a formula that produces each of the following sequences:

1. $\{1, -1, 1, -1, 1, \dots\}$
2. $\{-1, 4, -9, 16, -25, \dots\}$
3. $\{4, 2, 1, 1/2, 1/4, \dots\}$

The Hierarchy of Growth

Order the following sequences from smallest to largest as $n \to \infty$:

$$n!, \ln(\ln(n)), 3n+5, 7, n^{0.0001}, 0.8^n, e^n/200, n^n, \sqrt{n}, \sqrt{9n^2+3n+2}, \ln(n), 1.01^n, n^{100} - 2$$

Determining Convergence of a Sequence

Determine whether each of the following sequences has a limit of 0 or ∞:

1. $e^n/n!$
2. $n^2/\ln(n)$
3. $\sqrt{n}/1.01^n$
4. $\ln(n)/\ln(\ln(n))$
5. e^n/n^e
6. $n^2/\sqrt{n^5+2}$
7. $\frac{n^5 + 10\ln(n)}{5n^3 + 1.01^n}$
8. $\frac{3n + 5 + \ln\ln n}{n!}$
9. $\frac{1.05^n + n^3}{1.04^n + \sqrt{n}}$
10. $\frac{\sqrt{n^3+1}(n+1)^3}{(n + \ln n)^3}$
11. $\frac{e^n + e^{-n}}{2n}$
12. $\frac{n \ln\ln n}{1 + \ln n}$
13. $\frac{e^n \ln n + \sqrt{n}}{n!}$
14. $\frac{1.01^n}{n^{1.01}}$

Each of the following sequences has a positive finite limit. Find the limit:

1. $\sqrt{n^2+2n}-n$
2. $\sqrt{n^2+5n+1}-n$
3. $\sqrt{4n^2+6n+3}-2n$
4. $n\ln(1+1/n)$
5. $n\sin(1/n)$
6. $(1+1/n)^n$
7. $n(\ln(n+1)-\ln(n))$
8. $n\sqrt{n^2+1}-n^2$
9. $n(e^{1/n}-1)$

True or False?

If true, explain your reasoning. If false, find a counterexample.

1. If $\{a_n\}$ converges, then $\lim_{n\to\infty} a_n = 0$.
2. If $\{a_n\}$ diverges, then $\lim_{n\to\infty} a_n \neq 0$.
3. If $\{a_n\}$ converges, then there is an N such that $n \geq N$ implies $|a_n - a_{n+1}| < N$.
4. If $\{a_n\}$ diverges, then for any $\epsilon > 0$ there is an N such that $n \geq N$ implies $|(\lim_{n\to\infty} a_n) - a_n| > \epsilon$.
5. If $\lim_{n\to\infty} a_n = 0$ but $\{b_n\}$ diverges, then the sequence $\{a_n b_n\}$ diverges.
6. If $\{a_n\}$ and $\{b_n\}$ both converge, then $\{a_n + b_n\}$ converges.
7. If $\{a_n\}$ and $\{b_n\}$ both converge, then $\{a_n b_n\}$ converges.
8. If $\{a_n\}$ converges, then $\lim_{n\to\infty} a_n/n = 0$.
9. For every $\{a_n\}$ there is some $\{b_n\}$ such that $\{a_n b_n\}$ diverges.
10. Every bounded sequence is convergent.
11. Every bounded convergent sequence is monotonic.
12. If f is a function and $\lim_{n\to\infty} a_n = L$, then $f(L) = \lim_{n\to\infty} f(a_n)$.
13. If f is a continuous function and $\lim_{n\to\infty} a_n = L$, then $f(L) = \lim_{n\to\infty} f(a_n)$.
14. If $\{a_n\}$ converges, then $\{a_n^2\}$ converges.

Bonus

1. Prove that $\lim_{n\to\infty} \ln(n)/n^\epsilon = 0$ for any $\epsilon > 0$.
2. Prove that $\lim_{n\to\infty} k^n/n! = 0$ for any constant $k > 0$.