11.1/11.2: Sequences/Series Review

Monday, February 23

Speed Round

1. $\lim_{n\to\infty} 2^n/n^2$
2. $\lim_{n\to\infty} n/\ln(n)$
3. $\lim_{n\to\infty} e^n/n!$
4. $\lim_{n\to\infty} n^{100}/n!$
5. $\lim_{n\to\infty} \ln(n)/\ln(\ln(n))$
6. $\lim_{n\to\infty} \ln(n)/n^{0.0001}$
7. $\lim_{n\to\infty} \sin(n)$
8. $\lim_{n\to\infty} \sin^2(n) + \cos^2(n)$
9. $\lim_{n\to\infty} \dfrac{n^2+3n+2}{3n^2+2n+1}$
10. $\lim_{n\to\infty} \dfrac{n+\ln n}{\sqrt{n^2+\ln n}}$
11. $\lim_{n\to\infty} \dfrac{1.01^n+n^2}{0.95^n+n^5}$
12. $\lim_{n\to\infty} 0.9999^n$
13. $\lim_{n\to\infty} (-1)^n$
14. $\lim_{n\to\infty} \dfrac{e^n+n}{e^{2n}}$
15. $\lim_{n\to\infty} \dfrac{n\sqrt{n+1}}{\sqrt{n^3+1}}$
16. $\sum_{n=0}^{\infty} \pi^n$
17. $\sum_{n=0}^{\infty} (1/\pi)^n$
18. $\sum_{n=0}^{\infty} 5/2^n$
19. $\sum_{n=0}^{\infty} (5/2)^n$
20. $\sum_{n=0}^{\infty} 1/n$

Some Computation Required

1. $\lim_{n\to\infty} \sqrt{n^2-n} - n$
2. $\lim_{n\to\infty} \sqrt{n^2+1} - n$
3. $\lim_{n\to\infty} n(\cos(1/n) - 1)$
4. $\lim_{n\to\infty} n^2(\cos(1/n) - 1)$
5. Prove that if $\epsilon > 0$ then $\lim_{n\to\infty} \ln(n)/n^{\epsilon} = 0$.
6. $\sum_{n=1}^{\infty} 3^{n+2}/4^n$
7. $\sum_{n=1}^{\infty} 2^{n-2}/5^{n+1}$

Monotone Convergence Theorem

If $\{a_n\}$ is monotonic and bounded, then $\lim_{n\to\infty} a_n$ exists.

1. Draw a picture illustrating the Monotone Convergence Theorem.
2. True or False: $a_n = \ln(n)$ is monotonic, so the MTC implies that $\lim_{n\to\infty} a_n$ exists.
3. True or False: $s_n = \sum_{i=1}^{n} 1/i^2$ is monotonic and bounded above by 2, so the MTC implies that $\lim_{n\to\infty} a_n = 2$.
4. True of False: $a_n = 1 + 1/n$ is decreasing and bounded above by 2, so the MTC implies that $\lim_{n\to\infty} a_n$ exists.

5. If D_n is the world record in the 100-meter dash as of the year n (say, for $n \geq 1900$), what (if anything) does the MTC say about D_n?

6. A North-Going Zax (which only goes north) and a South-Going Zax (which only goes south) are on a collision path. What does the MTC say about the two Zax? Do they necessarily bump into each other?

Back to Polynomials

1. For what values of x does $\lim_{n\to\infty} x^n = 0$ hold?

2. Sketch the graphs of the functions $f(x) = x^{100}, g(x) = x^{101}, h(x) = 0$. When are the first two "good" approximations of the third?

3. For what values of x does $\sum_{n=0}^{\infty} x^n = 1/(1-x)$ hold?

4. Sketch the graphs of the functions $f(x) = 1+x+x^2+\ldots x^{50}, g(x) = 1+x+x^2+\ldots+x^{51}, h(x) = 1/(1-x)$. When are the first two "good" approximations of the third?

5. For what values of x does $\lim_{n\to\infty}(x/2)^n = 0$ hold?

6. For what values of x does $\sum_{n=0}^{\infty}(x/2)^n = 1/(1-x/2)$ hold?

7. What is $\sum_{n=0}^{\infty} x^{2n}$, when $|x| < 1$?