9.3: Separable Equations

Friday, April 10

Review

1. Show that $y = Ae^{2x} + Be^{-2x}$ satisfies the differential equation $y'' = 4y$ for any constants A, B.
2. Show that $y = \ln x$ satisfies the differential equation $y' = e^{-y}$.
3. Suppose a particle's velocity is described by $\frac{dx}{dt} = \sin x$. If $x(0) = 1$, what will happen to the particle as $t \to \infty$?
4. What if $x(0) = 2\pi$? What if $x(0) = -1$?
5. Sketch a direction field for the $\frac{dx}{dt} = \sin x$, including lines where $\frac{dx}{dt} = 0$.

Separable Equations

Find the solution of the differential equation that satisfies the given initial condition.

1. $\frac{dy}{dx} = y, y(0) = 2$
2. $\frac{dy}{dx} = xy, y(0) = 1$
3. $\frac{dy}{dx} = (1 + y)/x, y(0) = 3$
4. $\frac{dy}{dx} = 1 + x + y + xy, y(0) = -1$
5. $\frac{dy}{dx} = e^{y+x}, y(0) = 2$
6. $\frac{dy}{dx} = y + x, y(0) = 0$ (Hint: Substitute $u = y + x$)

Back to Air Resistance

Say we throw a pillow off a cliff. Acceleration due to gravity is constant, but air resistance is greater the faster the pillow moves. For this reason, we'll model the acceleration on the pillow as

$$a = \frac{dv}{dt} = -10 - v.$$

1. Find an equation for $v(t)$, assuming the initial velocity of the pillow is zero.
2. Find an equation for $y(t)$, the height of the pillow over time.
3. What happens to $v(t)$ and $y(t)$ as $t \to \infty$? Assume the cliff overlooks an infinite abyss.

Melting Snowball

A spherical snowball sits in the hot sun. Since only the surface of the snowball is exposed to the sunlight, assume that the snowball melts at a rate proportional to its surface area. After 1 hour, the snowball is half its original size.

1. Set up a differential equation in terms of volume, surface area, and time.
2. Put the previous equation in terms of time and the radius of the snowball.
3. How long does the snowball take to melt entirely?

Spreading Rumor

Model the rate at which a rumor spreads with $\frac{dP}{dt} = P(1 - P)$, where P is the perecentage of people who have heard the rumor.

1. Solve for $P(t)$ where $P(0) = 1/10$.
2. Check that this equation qualitatively behaves in the manner you would expect (how do you expect it to behave?)