11.8: Power Series!

Wednesday, March 11

Speed Round

Determine whether each of the following series converges or diverges.

1. $\sum_{n=1}^{\infty} \frac{1}{n}$
2. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$
3. $\sum_{n=1}^{\infty} \frac{2^n}{n!}$
4. $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$
5. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$
6. $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\ln n}$
7. $\sum_{n=1}^{\infty} \frac{n^2 \ln n}{4^n}$
8. $\sum_{n=1}^{\infty} \frac{n^5 5^n}{n!}$
9. $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^3}$
10. $\sum_{n=1}^{\infty} (-1)^n$
11. $\sum_{n=1}^{\infty} \frac{3^n}{n^2+\sqrt{n}}$
12. $\sum_{n=1}^{\infty} \frac{n!}{30^n}$

Interval of Convergence

How to tell at a glance what the interval of convergence is:

1. If $n!$ appears, ignore everything else. $R = 0$ or $R = \infty$.
2. If not, write your series (if possible) as $\sum_{n=1}^{\infty} A(n) \cdot (x-a)^n \cdot r^n$ where r^n is the part that increases exponentially and $A(n)$ is everything else.
3. $R = 1/|r|$. The interval of convergence is $(a-R, a+R)$, except maybe for the endpoints.
4. If $\sum_{n=1}^{\infty} A(n)$ converges absolutely, the interval is $[a-R, a+R]$.
5. If $\sum_{n=1}^{\infty} A(n)$ converges conditionally the interval will be one-sided (either $(a-R, a+R]$ or $[a-R, a+R)$).
6. If $\lim_{n\to\infty} A(n) \neq 0$, the interval is $(a-R, a+R)$.

Find the intervals of convergence of the following functions:

1. $\sum_{n=1}^{\infty} x^n$
2. $\sum_{n=1}^{\infty} \frac{x^n}{n}$
3. $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$
4. $\sum_{n=1}^{\infty} \frac{(x-3)^n}{2^n}$
5. $\sum_{n=1}^{\infty} \frac{(x-5)^n}{n \cdot 3^n}$
6. $\sum_{n=1}^{\infty} (-2)^n \frac{(x+4)^n}{\sqrt{n}}$
7. $\sum_{n=1}^{\infty} 5^n \frac{(x+3)^n}{n!}$
8. $\sum_{n=1}^{\infty} (x+7)^n \frac{n}{2^n}$
9. $\sum_{n=1}^{\infty} n!(x-1)^n$
10. $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n(-4)^n}$
11. $\sum_{n=1}^{\infty} \frac{x^n}{n!}$
12. $\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{n\sqrt{n}}$

Power Series Arithmetic!

1. $e^x =$
2. $\sin x =$
3. $\cos x =$
4. $\dfrac{1}{1-x} =$

Rewrite the power series above in sigma notation.

Add and Multiply!

1. $5e^x =$
2. $\sin x + \cos x =$

Compose!

1. $e^{-x^2} =$
2. $\sin(2x) =$
3. $\dfrac{1}{1-3x} =$
4. $\dfrac{1}{1+x^2} =$
5. $\dfrac{1}{5-x} =$
6. $\dfrac{1}{2+3x} =$

Differentiate and Integrate!

1. $\frac{d}{dx} e^x =$
2. $\frac{d}{dx} \sin x =$
3. $\int e^x =$
4. $\ln(1+x) = \int \frac{1}{1+x} =$
5. $\arctan x = \int \frac{1}{1+x^2} =$
6. $\int \sin x =$

. . . Multiply and Divide?

Verify the following:

1. $\sin(2x) = 2 \sin x \cos x$
2. $e^x e^{-x} = 1$
3. $\sin^2 x + \cos^2 x = 1$
4. Find $1/\cos(x)$ by solving $(a_0 + a_1 x + a_2 x^2 + \ldots)\cos(x) = 1$ term-by-term.

Invert??

1. Find $\arcsin(x)$ by solving $P(\sin x)$ term-by-term.
2. Find $\sqrt{1+x}$ by solving $P(x)^2 = 1 + x$.