11.8-11.9: Review

Monday, March 16

Intervals of Convergence

Find the interval of convergence of each of the following series:

1. $\sum_{n=1}^{\infty} n!x^n$
2. $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^2}$
3. $\sum_{n=1}^{\infty} \frac{(3x+1)^n}{n}$
4. $\sum_{n=1}^{\infty} \frac{x^n}{5^n\sqrt{n}}$
5. $\sum_{n=1}^{\infty} \frac{(x+5)^n}{n!}$
6. $\sum_{n=1}^{\infty} \frac{2^n(x-2)^n}{3^n}$
7. $\sum_{n=1}^{\infty} \frac{(x+1)^n}{n^2}$
8. $\sum_{n=1}^{\infty} \frac{(5-4x)^n}{n}$
9. $\sum_{n=1}^{\infty} \frac{(x-3)^n}{2^n\sqrt{n}}$

Find functions with the following intervals of convergence:

1. $[-1, 1]$
2. $[-1, 1)$
3. $(-1, 1)$
4. $(-1, 1]$
5. $(3, 5)$
6. $[-1, 6]$
7. $[2, 4)$
8. $\{1\}$
9. $(-\infty, \infty)$

A certain power series converges at $x = -1$ and diverges at $x = 5$. For each of the following values of a, decide whether it is possible for the power series to be centered at $x = a$:

1. $a = -3$
2. $a = -1$
3. $a = 1$
4. $a = 2$
5. $a = 3$
6. $a = 7$

Power Series

Write the first few terms of each of the following series (centered at $x = 0$). Write them in sigma notation. What are their intervals of convergence?

1. $e^x =$
2. $\sin x =$
3. $\cos x =$
4. $\arctan x =$
5. $\frac{1}{1-x} =$
6. $\ln(1+x) =$
7. $e^{2x} =$
8. $\frac{1}{1+x} =$
9. $\cos(-x) =$
10. $\sin(2x) =$
11. $\frac{1}{1+x^2} =$
12. $\sin^2(x) =$

Verify the following identities:

1. $\int \sin(2x)\,dx = \dfrac{-1}{2}\cos(2x)$
2. $\ln(1-x^2) = \ln(1+x) + \ln(1-x)$
3. $\dfrac{d}{dx}\dfrac{1}{1-x} = \left(\dfrac{1}{1-x}\right)^2$

Conceptual

1. The Taylor series for $\dfrac{1}{1-x}$ centered at $x=0$ is $1+x+x^2+x^3+\ldots$ What is the value of the function at $x=-1/2$? $x=2$? $x=1$? Can we use the power series to evaluate the function at these points?

2. The Taylor series for $\sin(x)$ at $x=0$ is $x-x^3/3!+x^5/5!-\ldots$ Can we use this series to find $\sin(10000)$? Is this a good idea? Come up with another way to estimate $\sin(10000)$.

3. The Taylor series for $\tan(x)$ at $x=0$ is fairly complicated. What do you think its interval of convergence will be? Why?

4. What if we center the Taylor series for $\tan(x)$ at $x=1$?