11.8-11.9: Review

Monday, March 16

Intervals of Convergence

Find the interval of convergence of each of the following series:

1. $\sum_{n=1}^{\infty} n!x^n$: $\{0\}$
2. $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^2}$: $[1,3]$
3. $\sum_{n=1}^{\infty} \frac{(3x+1)^n}{n}$: $[-2/3,0)$
4. $\sum_{n=1}^{\infty} \frac{x^n}{5^n\sqrt{n}}$: $[-5,5)$
5. $\sum_{n=1}^{\infty} \frac{(x+5)^n}{n!}$: $(-\infty,\infty)$
6. $\sum_{n=1}^{\infty} \frac{2^n(x-2)^n}{3^n}$: $(1/2,7/2)$
7. $\sum_{n=1}^{\infty} \frac{(x+1)^n}{n^2}$: $[-2,0]$
8. $\sum_{n=1}^{\infty} \frac{(5-4x)^n}{n}$: $(2,3/2]$
9. $\sum_{n=1}^{\infty} \frac{(x-3)^n}{2^n\sqrt{n}}$: $[1,5)$

Find functions with the following intervals of convergence:

1. $[-1,1]$: $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$
2. $[-1,1)$: $\sum_{n=1}^{\infty} \frac{x^n}{n}$
3. $(-1,1)$: $\sum_{n=1}^{\infty} x^n$
4. $(-1,1]$: $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2}$
5. $(3,5)$: $\sum_{n=1}^{\infty} (x-4)^n$
6. $[-1,6]$: $\sum_{n=1}^{\infty} \frac{2^n(x-5/2)^n}{n^2\cdot 7^n}$
7. $[2,4)$: $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$
8. $\{1\}$: $\sum_{n=1}^{\infty} n!(x-1)^n$
9. $(-\infty,\infty)$: $\sum_{n=1}^{\infty} \frac{x^n}{n!}$

A certain power series converges at $x=-1$ and diverges at $x=5$. For each of the following values of a, decide whether it is possible for the power series to be centered at $x=a$:

1. $a=-3$ yes
2. $a=-1$ yes
3. $a=1$ yes
4. $a=2$ yes (this is the largest possible value for the center)
5. $a=3$ no
6. $a=7$ no

Power Series

Write the first few terms of each of the following series (centered at $x=0$). Write them in sigma notation. What are their intervals of convergence?

1. $e^x = 1 + x + x^2/2! + x^3/3! + \ldots$ IOC $= (-\infty,\infty)$
2. $\sin x = x - x^3/3! + x^5/5! - \ldots$ IOC $= (-\infty,\infty)$
3. $\cos x = 1 - x^2/2! + x^4/4! - \ldots$ IOC $= (-\infty,\infty)$
4. $\arctan x = x - x^3/3 + x^5/5 - \ldots$ IOC $= [-1,1]$
5. $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$ IOC = (-1,1)
6. $\ln(1+x) = x - x^2/2 + x^3/3 - x^4/4 + \ldots$ IOC = (-1,1]
7. $e^{2x} = 1 + (2x) + (2x)^2/2! + (2x)^3/3! + \ldots$ IOC $= (-\infty,\infty)$
8. $\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \ldots$ IOC = (-1,1)
9. $\cos(-x) = 1 - x^2/2! + x^4/4! - \ldots$ IOC $= (-\infty,\infty)$

10. $\sin(2x) = 2x - (2x)^3/3! + (2x)^5/5! - \ldots$ IOC $= (-\infty, \infty)$
11. $\dfrac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \ldots$ IOC $= (-1, 1)$
12. $\sin^2(x) = x^2 - x^4/3 + 2x^6/45 - \ldots$ IOC $= (-\infty, \infty)$

Verify the following identities:

1. $\displaystyle\int \sin(2x)\, dx = \frac{-1}{2}\cos(2x)$
2. $\ln(1 - x^2) = \ln(1 + x) + \ln(1 - x)$
3. $\dfrac{d}{dx}\dfrac{1}{1-x} = \left(\dfrac{1}{1-x}\right)^2$

Conceptual

1. The Taylor series for $\dfrac{1}{1-x}$ centered at $x = 0$ is $1 + x + x^2 + x^3 + \ldots$ What is the value of the function at $x = -1/2$? $x = 2$? $x = 1$? Can we use the power series to evaluate the function at these points?

2. The Taylor series for $\sin(x)$ at $x = 0$ is $x - x^3/3! + x^5/5! - \ldots$ Can we use this series to find $\sin(10000)$? Is this a good idea? Come up with another way to estimate $\sin(10000)$.

3. The Taylor series for $\tan(x)$ at $x = 0$ is fairly complicated. What do you think its interval of convergence will be? Why?

4. What if we center the Taylor series for $\tan(x)$ at $x = 1$?