Math 1B, Quiz 8

Monday, March 30

INSTRUCTIONS: There are nine questions on this test, worth 2 points apiece. Choose and answer SIX of these questions.

1. For each of the following series determine whether the series is divergent, conditionally convergent, or absolutely convergent. You DO NOT have to show your work.

   (a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{1}{n\sqrt{n+1}}$–ABS

   (b) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{\sqrt{n+1}}{n}$–CC

   (c) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{n}{\sqrt{n+1}}$–DIV

2. Mark each question as True or False. If the answer is true, explain why (concisely). If the answer is false, give a counterexample.

   (a) If the sequence $\{a_n\}_{n=1}^{\infty}$ is monotonic and $a_n > 0$ for all $n \in \mathbb{N}$, then $\{a_n\}_{n=1}^{\infty}$ converges.
   False: $a_n = n$ serves as a counterexample.

   (b) If $\sum_{n=1}^{\infty} a_n$ converges conditionally then $\sum_{n=1}^{\infty} a_n\sqrt{n}$ diverges.
   False: $a_n = (-1)^n/n$ serves as a counterexample.

3. Up to the x^5 term, find the Taylor series around $x = 0$ for

$$f(x) = (1 - x^2)\sin(x)$$

$f(x) = (1 - x^2)(x - x^3/6 + x^5/120 - \ldots) = x - 7x^3/6 + 21x^5/120 - \ldots$ (or $7x^5/40$)

4. Find the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(2-3x)^n}{\sqrt{n}}$$

$$\begin{aligned} \lim_{n\to\infty} |a_{n+1}|/|a_n| &= \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+1}} \frac{|2-3x|^{n+1}}{|2-3x|^n} \\ &= |2-3x| \end{aligned}$$

So for the series to converge, we want

$$\begin{aligned} |2-3x| &< 1 \\ |2/3 - x| &< 1/3 \\ -1/3 < x - 2/3 &< 1/3 \\ 1/3 < x &< 1 \end{aligned}$$

Then checking the endpoints gives that the interval of convergence is $(1/3, 1]$.

5. Answer True or False. You DO NOT have to show your work.

(a) If $\sum_{n=1}^{\infty} a_n$ converges absolutely then the radius of convergence of $\sum_{n=1}^{\infty} a_n x^n$ is strictly greater than 1.

False: $a_n = 1/n^2$ converges absolutely but $\sum_{n=1}^{\infty} x^n/n^2$ has a radius of convergence of 1.

(b) If the radius of convergence of $\sum_{n=1}^{\infty} a_n x^n$ is strictly greater than 1 then $\sum_{n=1}^{\infty} a_n$ converges absolutely.

True.